Functional Nets

Martin Odersky
EPFL

ESOP 2000, Berlin

What's a Functional Net?

- Functional nets arise out of a fusion of key ideas of functional programming and Petri nets.
- Functional programming: Rewrite-based semantics with function application as the fundamental computation step.
- Petri nets: Synchronization by waiting until all of a given set of inputs is present, where in our case

  input = function application.

- A functional net is a concurrent, higher-order functional program with a Petri-net style synchronization mechanism.
- Theoretical foundation: Join calculus.

Thesis of this Talk

Functional nets are a simple, intuitive model

of { imperative / functional / concurrent } programming.

Functional nets combine well with OOP.

Elements

Functional nets have as elements:

- functions
- objects
- parallel composition

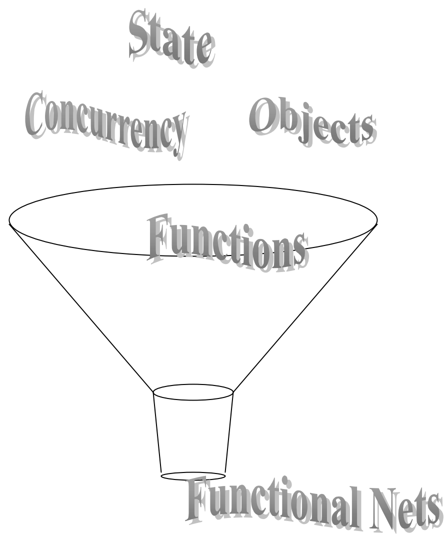
They are presented here as a calculus and as a programming notation.

Calculus: (Object-based) join calculus

Notation: Funnel[1] (alternatives are Join or JoCAML)

[1] "Funnel" is still named "Silk" in the paper; we changed the name because of the similarity in pronounciation to "Cilk", the concurrent C dialect.

The Principle of a Funnel



Stage 1: Functions

- A simple function definition:

```
def gcd (x, y) =
  if (y == 0) x
  else gcd (y, x % y)
```

- Function definitions start with **def**.
- Operators as in C/Java.
- Usage:

```
val x = gcd (a, b)
print (x * x)
```

- Call-by-value: Function arguments and right-hand sides of **val** definitions are always evaluated.

Stage 2: Objects

- One often groups functions to form a single value. Example:

```
def makeRat (x, y) = {
   val g = gcd (x, y)
   {  def numer   =   x / g
      def denom   =   y / g
      def add r   =   makeRat (
                        numer * r.denom + r.numer * denom,
                        denom * r.denom)
      ...
   }
}
```

- This defines a record with functions numer, denom, add, ...
- We identify: Record = Object, Function = Method
- For convenience, we admit parameterless functions such as numer.

Functions + Objects Give Algebraic Types

- Functions + Records can encode algebraic types
  - Church Encoding
  - Visitor Pattern
- Example: Lists are represented as records with a single method, match.
- match takes as parameter a *visitor* record with two functions:

```
{ def Nil = ...
  def Cons (x, xs) = ... }
```

- match invokes the Nil method of its visitor if the List is empty, the Cons method if it is nonempty.

Lists

- Here is an example how match is used.

```
def append (xs, ys) =
  xs.match {
    def Nil = ys
    def Cons (x, xs1) = List.Cons (x, append (xs1, ys))
  }
```

- It remains to explain how lists are constructed.

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Lists

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def append (xs, ys) =
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  }
```

- It remains to explain how lists are constructed.

- We wrap definitions for Nil and Cons constructors in a List "module". They each have the appropriate implementation of match.

```
val List = {
  def Nil            =   { def match v = ??? }
  def Cons (x, xs)   =   { def match v = ??? }
}
```

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Lists

- Here is an example how match is used.

```
def append (xs, ys) =
  xs.match {
    def Nil = ys
    def Cons (x, xs1) = List.Cons (x, append (xs1, ys))
  }
```

- It remains to explain how lists are constructed.
- We wrap definitions for Nil and Cons constructors in a List "module". They each have the appropriate implementation of match.

```
val List = {
  def Nil           =   { def match v = v.Nil }
  def Cons (x, xs)  =   { def match v = v.Cons (x, xs) }
}
```

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Stage 3: Concurrency

- Principle :
  - Function calls model *events.*
  - & means *conjunction* of events.
  - = means left-to-right rewriting.
  - & can appear on the right hand side of a = (*fork*) as well as on the left hand side (*join*).
- Analogy to Petri-Nets :

call	≈	place
equation	≈	transition

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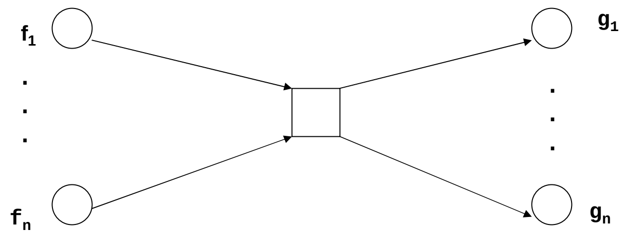
f_1 & ... & f = g_1 & ... & g_n

corresponds to



- Functional Nets are more powerful:
  - parameters,
  - nested definitions,
  - higher order.

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Example : One-Place Buffer

Functions : put, get (external)
empty, full (internal)

Definitions :

```
def put x & empty  =  () & full x
    get   & full x =  x  & empty
```

Usage :

```
val x = get ; put (sqrt x)
```

- An equation can now define more than one function.

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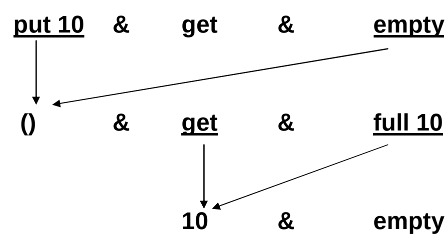
Rewriting Semantics

- A set of calls which matches the left-hand side of an equation is replaced by the equation's right-hand side (after formal parameters are replaced by actual parameters).
- Calls which do not match a left-hand side block until they form part of a set which does match.
- Example:



Objects and Joins

- We'd like to make a constructor function for one-place buffers.
- We could use tuples of methods:

```
def newBuffer = {
   def put x &  empty   =   ()  &  full x,
       get   &  full x  =   x   &  empty
   (put, get)  &  empty
}
val (bput, bget) = newBuffer ; ...
```

- But this quickly becomes combersome as number of methods grows.
- Usual record formation syntax is also not suitable
  - we need to hide function symbols
  - we need to call some functions as part of initialization.

Qualified Definitions

- Idea: Use qualified definitions:

```
def newBuffer = {
   def this.put x &  empty  =  ()  &  full x,
       this.get   &  full x  =  x   &  empty
   this & empty
}
val buf = newBuffer ; ...
```

- Three names are defined in the local definition:

this	- a record with two fields, get and put.
empty	- a function
full	- a function

- this is returned as result from newBuffer; empty and full are hidden.

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- The choice of this as the name of the record was arbitrary; any other name would have done as well.
- We retain a conventional record definition syntax as an abbreviation, by inserting implicit prefixes. E.g.

```
{ def numer = x / g
  def denom = y / g }
```

is equivalent to

```
{ def r.numer = x / g, r.denom = y / g ; r }
```

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Mutable State

- A *variable* (or reference cell) with functions

  read, write (external)
  state (internal)

  is created by the following function:

```
def newRef init = {
   def this.read    & state x  =  x  &  state x,
       this.write y &  state x  =  () &  state y
   this & state init
}
```

- Usage:

```
val r = newRef 0 ; r.write (r.read + 1)
```

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Stateful Objects

- An object with *methods* $m_1,...,m_n$ and *instance variables* $x_1,...,x_k$ can be expressed such :

def this.m_1 & state ($x_1,...,x_k$) = ... ; state ($y_1,...,y_k$),
:
:
this.m_n & state ($x_1,...,x_k$) = ... ; state ($z_1,...,z_k$);

this & state ($init_1,..., init_k$)

« Result » « initial state »

- The encoding enforces *mutual exclusion*, makes the object into a *monitor.*

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Synchronization

- Functional nets are very good at expressing many process synchronization techniques.
- Example: Readers/Writers Synchronization.
- Specification: Implement operations startRead, startWrite, endRead, endWrite such that:
  - there can be multiple concurrent reads,
  - there can be only one write at one time,
  - reads and writes are mutually exclusive,
  - pending write requests have priority over pending reads, but don 't preempt ongoing reads.

First Version

- Introduce two auxiliary state functions
  readers n - the number of *active* reads
  writers n - the number of *pending* writes
- Equations:

```
def startRead   & writers 0  =  startRead1,
    startRead1  & readers n  =  () & writers 0 & readers (n+1),

    startWrite  & writers n  =  startWrite1 & writers (n+1),
    startWrite1 & readers 0  =  (),

    endRead     & readers n  =  readers (n-1),
    endWrite    & writers n  =  writers (n-1) & readers 0

readers 0 & writers 0
```

- Note the almost-symmetry between startRead and startWrite, which reflects the different priorities of readers and writers.

Final program

- The previous program was is not yet legal Funnel since it contained numeric patterns.
- We can get rid of value patterns by partitioning state functions.

```
def startRead   & noWriters   =  startRead1,
    startRead1 & noReaders   =  () & noWriters & readers 1,
    startRead1 & readers n   =  () & noWriters & readers (n+1),

    startWrite  & noWriters   =  startWrite1 & writers 1,
    startWrite  & writers n   =  startWrite1 & writers (n+1),
    startWrite1 & noReaders  =  (),

    endRead    & readers n   =  if (n == 1) noReaders else (readers (n-1)),
    endWrite   & writers n   =  noReaders &
                                ( if (n == 1) noWriters else writers (n-1) )
noWriters & noReaders
```

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Summary : Concurrency

- Functional nets support an event-based model of concurrency.
- Channel based formalisms such as CCS, CSP or π - Calculus can be easily encoded.
- High-level synchronization à la Petri-nets.
- Takes work to map to instructions of hardware machines.
- Options:
  - Search patterns linearly for a matching one,
  - Construct finite state machine that recognizes patterns,
  - others?

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Foundations

- We now develop a formal model of functional nets.
- The model is based on an adaptation of join calculus (Fournet & Gonthier 96)
- Two stages: sequential, concurrent.

A Calculus for Functions and Objects

- Name-passing, continuation passing calculus.
- Closely resembles intermediate language of FPL compilers.

Syntax:

Names	x, y, z		
Identifiers	i, j, k	::=	x | i.x
Terms	M, N	::=	i j | **def** D ; M
Definitions	D	::=	L = M | D, D | 0
Left-hand Sides	L	::=	i x

Reduction:

def D, i x = M ; ... i j ... $\rightarrow$ **def** D, i x = M ; ... [j/x] M ...

A Calculus for Functions and Objects

- The ... ... dots are made precise by a *reduction context*.
- Same as Felleisen's evaluation contexts but there's no evaluation here.

Syntax:

Names	x, y, z		
Identifiers	i, j, k	::=	x | i.x
Terms	M, N	::=	i j | **def** D ; M
Definitions	D	::=	L = M | D, D | 0
Left-hand Sides	L	::=	i x
Reduction Contexts	R	::=	[ ] | **def** D ; R

Reduction:

def D, i x = M ; R[i j] $\rightarrow$ **def** D, i x = M ; R[[j/x]M]

Structural Equivalence

- Alpha renaming
- Comma is AC, with the empty definition 0 as identity

$$D_1, D_2 \equiv D_2, D_1$$
$$D_1, (D_2, D_3) \equiv (D_1,D_2), D_3$$
$$0, D \equiv D$$

Properties

- Name-passing calculus - every value is a (qualified) name.
- Mutually recursive definitions are built in.
- Functions with results are encoded via a CPS transform (see paper).
- Value definitions can be encoded:

  val x = M ; N ≡ **def** k x = N ; k M

- Tuples can be encoded:

  f (i, j) ≡ (**def** ij.fst () = i, ij.snd () = j ; f ij)

  f (x, y) = M ≡ f xy = (**val** x = xy.fst () ; **val** y = xy.snd () ; M)

A Calculus for Functions, Objects and Concurrency

Syntax:

Names	x, y, z		
Identifiers	i, j, k	::=	x | i.x
Terms	M, N	::=	i j | **def** D ; M | M & M
Definitions	D	::=	L = M | D, D | 0
Left-hand Sides	L	::=	i x | L & L
Reduction Contexts	R	::=	[ ] | **def** D ; R | R & M | M & R

Reduction:

def D, $i_1\ x_1$ & ... & $i_n\ x_n$ = M ; R [$i_1\ j_1$ & ... & $i_n\ j_n$]

→ **def** D, $i_1\ x_1$ & ... & $i_n\ x_n$ = M ; R [[j_1/x_1,...j_n/x_n] M]

Structural Equivalence

- Alpha renaming
- Comma is AC, with the empty definition 0 as identity:
- & is AC:

$$M_1, M_2 \equiv M_2, M_1$$
$$M_1, (M_2, M_3) \equiv (M_1,M_2), M_3$$

- Scope Extrusion:

(**def** D ; M) & N ≡ **def** D ; M & N

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Relation to Join Calculus

- Strong connections to join calculus.
  - \- Polyadic functions
  - \+ Records, via qualified definitions and accesses.
- Formulated here as a rewrite system, whereas original join uses a reflexive CHAM.
- The two formulations are equivalent.

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Conclusions

- Functional nets provide a simple, intuitive way to think about functional, imperative, and concurrent programs.
- They are based on join calculus.
- Mix-and-match approach: functions (+objects) (+concurrency).
- Close connections to
  - sequential FP (a subset),
  - Petri-nets (another subset),
  - π-Calculus (can be encoded easily).
- Functional nets admit a simple expression of object-oriented concepts.

State ofWork

Done :

- Design of Funnel,
- experimental Hindley/Miler style type system,
- First, dynamically typed, implementation (available from `http://lampwww.epfl.ch`).

- Current :
  - More powerful type System,
  - Efficient compilation strategies,
  - Encoding of objects
  - Funnel as a composition language in a Java environment.

  - Collaborators : Philippe Altherr, Matthias Zenger, Christoph Zenger (EPFL)
    Stewart Itzstein (Uni South Australia).