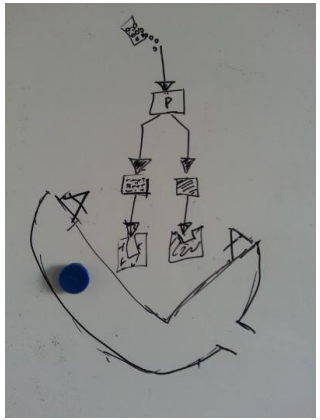
Fold-Based Fusion as a Library

A Generative Programming Pearl



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ÉCOLE POLYTECHNIQUE
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An Example

```
val people2Movies: List[(String, List[String])] = List(
  (“Sébastien”, List(“Hunger Games”, “Silver Linings Playbook”)),
  (“Eugene”, List(“Gattaca”, “Inside Man”)),
  (“Hubert”, List(“Silver Linings Playbook”, “Lost in Translation”)),
  (“Sandro”, List(“Lost in Translation”, “The Matrix”, “Pulp Fiction”)),
  (“Heather”, List(“Django Unchained”, “Tropic Thunder”, “Pulp Fiction”)),
  …
)
```

Question: How many people like each movie?

The Scala Way

```
def movieCount(people2Movies: List[(String, List[String])]): Map[String, Int] = {
  val flattened = for {
    (person, movies) <- people2Movies
    movie <- movies
  } yield (person, movie)
  val grouped = flattened groupBy (_._2)
  grouped map { case (movie, ps) => (movie, ps.size) }
}
```

An Optimized Way

```
def movieCount2(people2Movies: List[(String, List[String])]): Map[String, Int] = {
  var tmpList = people2Movies; val tmpRes: Map[String, Int] = Map.empty
  while (!tmpList.isEmpty) {
    val hd = tmpList.head; var movies = hd._2
    while (!movies.isEmpty) {
        val movie = movies.head
        if (tmpRes.contains(movie)) {
            tmpRes(movie) += 1
        } else tmpRes.update(movie, 1)
          movies = movies.tail
      }
    tmpList = tmpList.tail
  }
  tmpRes
}
```

Fusion (Deforestation)

- `movieCount` (more readable) -> `movieCount2` (no intermediate structures)
- Desirable properties of a fusion algorithm
  - should deforest as many operations as possible.
  - should be simple, elegant even.

Fusion in the Large

- Haskell
  - use built-in fusion algorithms.
  - use rewrite rule system.
- Scala
  - Scala Blitz
  - The Dotty Linker

In this Presentation

- Fusion as a Library (aka let's build it ourselves)
  - Fold-Based Fusion, as powerful as foldr/build Fusion.
  - Applies to producers.
  - Also works for partitioning and grouping functions.

The Gist

- Convert Data Structures to their CPS-encoded versions
  - composition over structures -> composition over functions

FoldLeft

- Partially evaluate function composition -> deforestation

Lightweight Modular Staging

FoldLeft

```
def foldLeft[A, S](ls: List[A])(z: S, comb: (S, A) => S): S = ls match {
  case Nil     => z
  case x :: xs => foldLeft(xs)(comb(z, x), comb)
}

def map[A, B](ls: List[A], f: A => B): List[B] = foldLeft[A, List[B]](ls)(
  Nil,
  (acc, elem) => acc :+ f(elem)
)
```

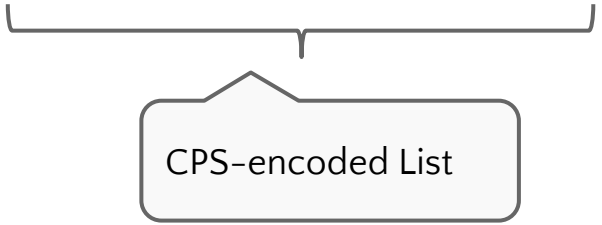
FoldLeft

```
def filter(ls: List[A], p: A => Boolean) = foldLeft[A, List[A]](ls)(
  Nil,
  (acc, elem) => if (p(elem)) acc :+ elem else acc
)

def flatMap[A, B](ls: List[A], f: A => List[B]) = foldLeft[A, List[B]](ls)(
  Nil,
  (acc, elem) => foldLeft[B, List[B]](f(elem))(
    acc,
    (acc2, elem) => acc2 :+ elem
  )
)
```

The Essence of FoldLeft

```
foldLeft[A, S]: List[A] => ((S, (S, A) => S) => S)
```



Abstracting over CPS-Encoded Lists

```
//foldLeft[A, S]: List[A] => ((S, (S, A) => S) => S)

type Comb[A, S] = (S, A) => S

abstract class CPSList[A] { self =>
  def apply[S](z: S, comb: Comb[A, S]): S
      …
}
```

The API of CPSList

```
//foldLeft[A, S]: List[A] => ((S, (S, A) => S) => S)
abstract class CPSList[A] { self =>

  …

  def map[B](f: A => B): CPSList[B] = …

  def filter(p: A => Boolean): CPSList[A] = …

  def flatMap(f: A => CPSList[B]): CPSList[B] = …

}

object CPSList {

  def fromList[A](ls: List[A]): CPSList[A] = …

  def fromRange(a: Int, b: Int): CPSList[Int] = …

}
```

The API of CPSList

```
def map[A, B](ls: List[A], f: A => B): List[B] = foldLeft[A, List[B]](ls)(
  Nil,
  (acc, elem) => acc :+ f(elem)
)

abstract class CPSList[A] { self =>
  ...
  def map[B](f: A => B) = new CPSList[B] {
    def apply[S](z: S, comb: Comb[B, S]) = self.apply(
      z,
      (acc: S, elem: A) => comb(acc, f(elem))
    )
  }
  ...
}
```

Using CPSList

```
def listExample(a: Int, b: Int) = (a to b).toList
  .flatMap(i => (1 to i).toList)
  .filter(_ % 2 == 1)
  .map(_ * 3).sum


def cpsListExample(a: Int, b: Int) = {
  val pipeline =
    CPSList.fromRange(a, b).flatMap(i => CPSList.fromRange(1 to i))
    .filter(_ % 2 == 1)
    .map(_ * 3)
  pipeline.apply[Int](0, (acc, x) => acc + x)
}
```

fold only applied at the very end

Welcome to Part II

- Convert Data Structures to their CPS-encoded versions
  - composition over structures -> composition over functions

FoldLeft

- Partially evaluate function composition -> deforestation

Lightweight Modular Staging

Partial Evaluation and Staging

- Partial evaluation
  - pre-evaluate parts of a program
  - residual program is specialized -> better performance

- Staging, aka. Multi-Stage Programming
  - Separate parts of the program in terms of evaluation
  - some parts are executed “now”
  - other parts are delayed to the next stage
  - => use staging for *controlled* partial evaluation

Partially Evaluating CPSList

```
def cpsListExample(a: Int, b: Int) = {
  val pipeline =
    CPSList.fromRange(a, b).flatMap(i => CPSList.fromRange(1 to i))
    .filter(_ % 2 == 1)
    .map(_ * 3)
  pipeline.apply[Int](0, (acc, x) => acc + x)
}
```

Partially Evaluating CPSList

```
def cpsListExample(a: Int, b: Int) = {
  val pipeline$1 =
    CPSList.fromRange(a, b).flatMap(i => CPSList.fromRange(1 to i))
    .filter(_ % 2 == 1)
  pipeline$1.apply[Int](0, (acc, x) => acc + x * 3)
}
```

Partially Evaluating CPSList

```
def cpsListExample(a: Int, b: Int) = {
  val pipeline$1$2 =
    CPSList.fromRange(a, b).flatMap(i => CPSList.fromRange(1 to i))
  pipeline$1$2.apply[Int](
    0,
    (acc, x) => if (x % 2 == 1) acc + x * 3 else acc
  )
}
```

Partially Evaluating CPSList

```
def cpsListExample(a: Int, b: Int) = {
  val pipeline$1$2$3 = CPSList.fromRange(a, b)
  pipeline$1$2$3.apply[Int](
    0,
    (acc, x) => CPSList.fromRange(1 to x).apply[Int](
      acc,
      (innerAcc, y) => if (y % 2 == 1) innerAcc + y * 3
                       else            innerAcc
    )
  )
}
```

Partially Evaluating CPSList

```
def cpsListExample(a: Int, b: Int) = {
  @tailrec
  def loop(a1: Int, b1: Int, tmpRes: Int) =
    if (a1 > b1) tmpRes
    else loop(a1 + 1, b1, innerLoop(1, a, tmpRes))
  @tailrec
  def innerLoop(i1: Int, i2: Int, tmpRes: Int) =
    if (i1 > i2) tmpRes
    else innerLoop(i1 + 1, i2,
      if (i1 % 2 == 1) tmpRes + i1 * 3 else tmpRes
    )
  loop(a, b, 0)
}
```

Partially Evaluating CPSList

```
def cpsListExample(a: Int, b: Int) = {
  var tmpRes: Int = 0; var i = a
  while (i <= b) {
    var j = 1
    while (j <= i) {
      if ((j % 2) == 1) { tmpRes += j * 3 }
      j += 1
    }
    i += 1
  }
  tmpRes
}
```

The Punchline

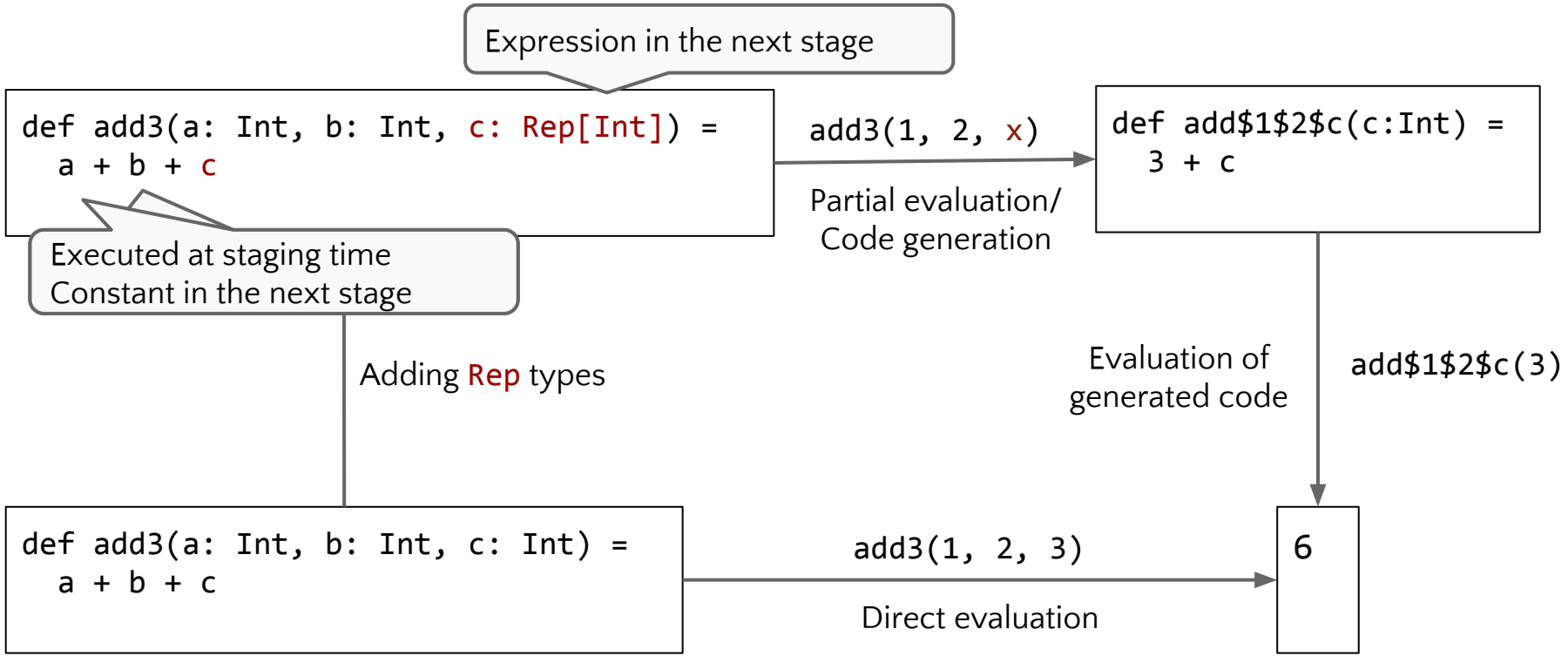
- Convert Data Structures to their CPS-encoded versions
  - composition over structures -> composition over functions

FoldLeft

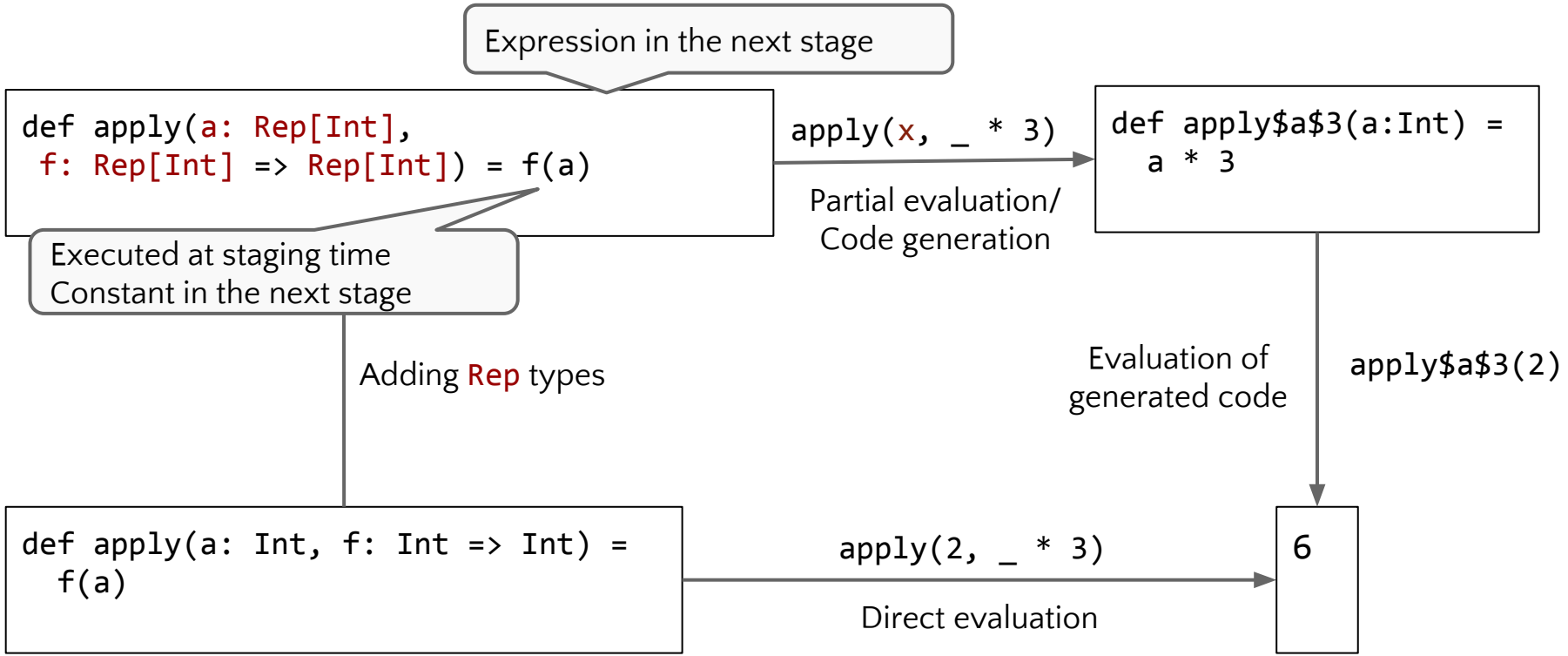
- Partially evaluate function composition -> deforestation

Lightweight Modular Staging

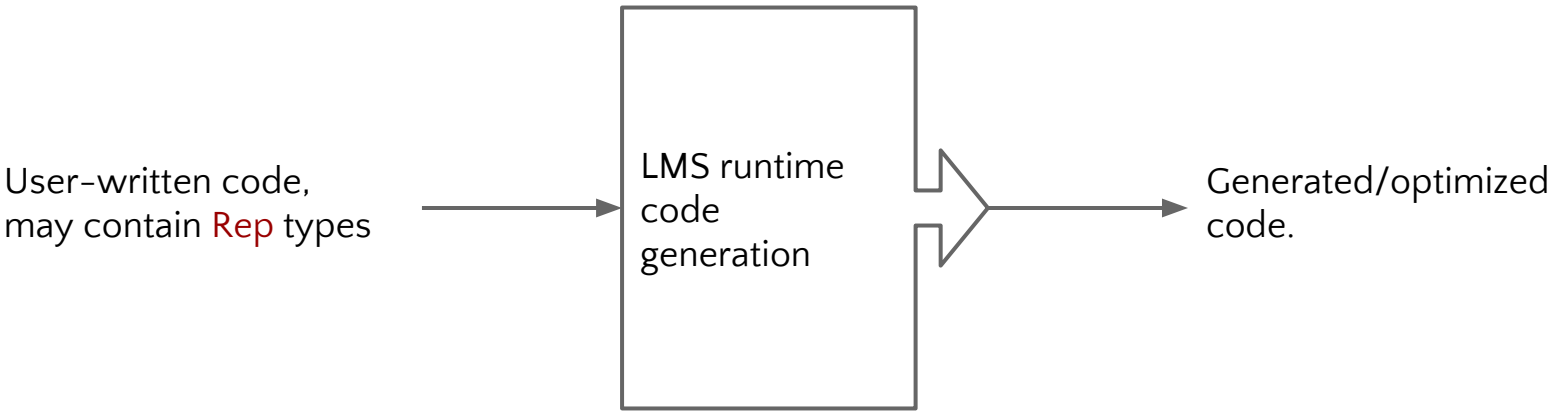
Partial Evaluation with LMS



Partial Evaluation with LMS: Functions



LMS


User-written code,
may contain Rep types
LMS runtime
code
generation
Generated/optimized
code.

Staging CPSList

```
foldLeft[A, S]: List[A] => ((S, (S, A) => S) => S)
```

Staging CPSList

```
stagedFoldLeft[A, S]: Rep[List[A]] =>
    ((Rep[S], (Rep[S], Rep[A]) => Rep[S]) => Rep[S])
```

Staging CPSList

```
type Comb[A, S] = (Rep[S], Rep[A]) => Rep[S]

abstract class CPSList[A] { self =>
  def apply[S](z: Rep[S], comb: Comb[A, S]): Rep[S]
      …
}
```

The API of Staged CPSList

```
abstract class CPSList[A] { self =>
  …
  def map[B](f: Rep[A] => Rep[B]): CPSList[B] = …
  def filter(p: Rep[A] => Rep[Boolean]): CPSList[A] = …
  def flatMap(f: Rep[A] => CPSList[B]): CPSList[B] = …
}

object CPSList {
  def fromList[A](ls: Rep[List[A]]): CPSList[A] = …
  def fromRange(a: Rep[Int], b: Rep[Int]): CPSList[Int] = …
}
```

The Rabbit out of the Hat

- Convert Data Structures to their CPS-encoded versions
  - composition over structures -> composition over functions

FoldLeft

- Partially evaluate function composition -> deforestation

Lightweight Modular Staging

- Multiple Producers

Multiple Element Producers

- So far: API contains only single element producers
- Next, partitioning and grouping:
  - Produce multiple elements.
  - We look at partitioning here.
  - Grouping: in the paper/talk to me later.

The Partition Function

```
def partition[A](ls: List[A], p: A => Boolean): (List[A], List[A]) =
  foldLeft[A, (List[A], List[A])](ls)(
    (Nil, Nil), {
    case ((trues, falses), elem) =>
      if (p(elem)) (trues ++ List(elem), falses)
      else          (trues, falses ++ List(elem))
})

val myList: List[Int] = ...
val (evens, odds) = partition(myList, (x: Int) => x % 2 == 0)
(evens map (_ * 2), odds map (_ * 3))
```

Staged Partition, a Naive Attempt

```
//as a method on CPSList
def partition(p: Rep[A] => Rep[Boolean]): (CPSList[A], CPSList[A]) = {
  val trues  = this filter p
  val falses = this filter (a => !p(a))
  (trues, falses)
}
```

Either: Keeping Things on One Pipeline

```
def partitionE[A](ls: List[A], p: A => Boolean): List[Either[A, A]] =
  ls map { elem => if (p(elem)) Left(elem) else Right(elem) }
```

Either = wrap an extra box

```
val myList: List[Int] = ...
val partitioned = partitionE(myList, (x: Int) => x % 2 == 0)
val mapped = partitioned map {
  case Left(x) => Left(x * 2)
  case Right(x) => Right(x * 3)
}
```

keeping things on one pipeline

```
foldLeft[Either[Int, Int], (List[Int], List[Int])](mapped)(
  (Nil, Nil), {
    case ((trues, falses), elem) =>
      elem.fold(x => (trues ++ List(x), falses), x => (trues, falses ++ List(x)))
 })
```

call to foldLeft delayed

Staged Partition, Bis

```
//as methods on CPSList
def partitionBis(p: Rep[A] => Rep[Boolean]): CPSList[Either[A, A]] =
  this map { elem =>
    if (p(elem)) left[A, A](elem)
    else         right[A, A](elem)
  }
```

constructors of instances of Rep [Either[A, A]]

`Rep[Either]` means boxes in generated code!

Remembering the Gist: CPSEither

```
abstract class CPSEither[A, B] {
  def apply[X](lf: A => X, rf: B => X): X
}

----> LMS ---->
abstract class CPSEither[A, B] {
  def apply[X](
    lf: Rep[A] => Rep[X],
    rf: Rep[B] => Rep[X]
  ): Rep[X]
 ...
}
```

More in the Paper on

- Grouping functions
  - CPS encoding of tuples
- LMS-specific implementation details
  - Code generation for conditional expressions

The Ecosystem of Fusion Algorithms

	Foldr/build, Staged CPSList	Unfoldr/destroy	Stream fusion
Producers	✓	✘	✓
Consumers	✘	✓	✓
Details	cannot handle zip-like functions	issues with filter, flatMap	fusion algorithm more involved, esp. for flatMap

Thank you!

https://github.com/manojo/staged-fold-fusion/

Staged CPSList

```
foldLeft[A, S]: List[A] => (S, (S, A) => S) => S

def fromList[A](ls: Rep[List[A]]) = new CPSList[A] {
  def apply[S](z: Rep[S], comb: Comb[A, S]): Rep[S] = {
    var tmpList = ls
    var tmp = z
    while (!tmpList.isEmpty) {
      tmp = comb(tmp, tmpList.head)
      tmpList = tmpList.tail
    }
    tmp
  }
}
```

The Staged CPSList API

```
//as methods of CPSList
def map[B](f: Rep[A] => Rep[B]) = new CPSList[B] {
  def apply[S](z: Rep[S], comb: Comb[B, S]) = self.apply(
    z,
    (acc: Rep[S], elem: Rep[A]) => comb(acc, f(elem))
  )
}
```

The Staged CPSList API

```
//as methods of CPSList
def filter(p: Rep[A] => Rep[Boolean]) = new CPSList[A] {
  def apply[S](z: Rep[S], comb: Comb[A, S]) = self.apply(
    z,
    (acc: Rep[S], elem: Rep[A]) =>
      if (p(elem)) comb(acc, elem) else acc
  )
}
```

The Staged CPSList API

```
//as methods of CPSList
def flatMap[B](f: Rep[A] => FoldLeft[B]) = new CPSList[B] {
  def apply[S](z: Rep[S], comb: Comb[B, S]) = self.apply(
    z,
    (acc: Rep[S], elem: Rep[A]) => f(elem)(acc, comb)
  )
}
```

An Example

```
def foldLeftExample(a: Rep[Int], b: Rep[Int]): Rep[Int] = {
  val fld = CPSList.fromRange(a, b)
  val flatMapped = fld flatMap {
     i => CPSList.fromRange(1, i)
  }
  val filtered = flatMapped filter (_ % 2 == 1)
  filtered.map(_ * 3).apply[Int](
     0, (acc, x) => acc + x
  )
}
```

An Example

```
def generatedFunction(x0:Int, x1:Int): Int = {
  var x2: Int = x0; var x3: Int = 0
  while (x2 <= x1) {
    val x7 = x3
    val x8 = x2
    var x9: Int = 1
    var x10: Int = x7
      ...
     val x26 = x10; x3 = x26
      val x28 = x8 + 1; x2 = x28
  }
  val x32 = x3
  x32
}
```

```
while (x9 <= x8) {
  val x14 = x10; val x15 = x9
  val x16 = x15 % 2; val x17 = x16 == 1
  val x20 = if (x17) {
    val x18 = x15 * 3
    val x19 = x14 + x18
    x19
  } else {
    x14
  }
  x10 = x20
  val x22 = x15 + 1
  x9 = x22
}
```

Staging = Multi-Stage Programming

- Separate parts of the program in terms of evaluation
  - some parts are executed “now”
  - other parts are delayed to the next stage
- Related concept: partial evaluation
  - pre-evaluate parts of a program
  - residual program is specialized -> better performance

Staging (LMS)

```
Rep[T => U]
```

Staged function. Code generation yields a function.

```
Rep[T] => Rep[U]
```

Unstaged function on staged types. Application inlines body of function. Generated code contains no function call.