STLC type derivation tree (1)

$$\dfrac{\dfrac{}{[a_2 \mapsto T_1\ ] \vdash a_2 : T_1}\ \text{T-Var}}{[] \vdash (\lambda a_0 . a_0) : (T_1 \Rightarrow T_1)}\ \text{T-Abs}$$

STLC type derivation tree (2)

$$\dfrac{\dfrac{\dfrac{}{[a_2 \mapsto (T_1 \Rightarrow T_1)\,] \vdash a_2 : (T_1 \Rightarrow T_1)}\ \text{T-Var}}{[] \vdash (\lambda a_0 . a_0) : ((T_1 \Rightarrow T_1) \Rightarrow (T_1 \Rightarrow T_1))}\ \text{T-Abs} \qquad \dfrac{\dfrac{}{[a_3 \mapsto T_1\,] \vdash a_3 : T_1}\ \text{T-Var}}{[] \vdash (\lambda a_0 . a_0) : (T_1 \Rightarrow T_1)}\ \text{T-Abs}}{[] \vdash ((\lambda a_0 . a_0)\ (\lambda a_0 . a_0)) : (T_1 \Rightarrow T_1)}\ \text{T-App}$$

STLC type derivation tree (3)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{[a_8 \mapsto T_3\ a_7 \mapsto (T_4 \Rightarrow T_5)\ a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash a_7 : (T_4 \Rightarrow T_5)}\ \text{T-Var} \quad \dfrac{}{[a_8 \mapsto T_3\ a_7 \mapsto (T_4 \Rightarrow T_5)\ a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash a_6 : (T_3 \Rightarrow T_4)}\ \text{T-Var}}{[a_8 \mapsto T_3\ a_7 \mapsto (T_4 \Rightarrow T_5)\ a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash (a_6\ a_8) : T_4}\ \text{T-App} \quad \dfrac{}{[a_8 \mapsto T_3\ a_7 \mapsto (T_4 \Rightarrow T_5)\ a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash a_8 : T_3}\ \text{T-Var}}{[a_8 \mapsto T_3\ a_7 \mapsto (T_4 \Rightarrow T_5)\ a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash (a_7\ (a_6\ a_8)) : T_5}\ \text{T-App}}{[a_7 \mapsto (T_4 \Rightarrow T_5)\ a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash (\lambda a_2.(a_7\ (a_6\ a_2))) : (T_3 \Rightarrow T_5)}\ \text{T-Abs}}{[a_6 \mapsto (T_3 \Rightarrow T_4)\,] \vdash (\lambda a_1.(\lambda a_2.(a_1\ (a_6\ a_2)))) : ((T_4 \Rightarrow T_5) \Rightarrow (T_3 \Rightarrow T_5))}\ \text{T-Abs}}{[] \vdash (\lambda a_0.(\lambda a_1.(\lambda a_2.(a_1\ (a_0\ a_2))))) : ((T_3 \Rightarrow T_4) \Rightarrow ((T_4 \Rightarrow T_5) \Rightarrow (T_3 \Rightarrow T_5)))}\ \text{T-Abs}$$

STLC type derivation tree debugging (1)

$$\dfrac{\dfrac{\dfrac{}{[a_3 \mapsto (T_1 \Rightarrow T_2)\,] \vdash a_3 : (T_1 \Rightarrow T_2)}\ \text{T-Var} \quad \dfrac{}{[a_3 \mapsto (T_1 \Rightarrow T_2)\,] \vdash a_3 : (T_1 \Rightarrow T_2)}\ \text{T-Var} \quad (T_1 \Rightarrow T_2) \neq T_1}{[a_3 \mapsto (T_1 \Rightarrow T_2)\,] \vdash (a_3\ a_3) : T_2}\ \text{T-App}}{[] \vdash (\lambda a_0.(a_0\ a_0)) : ((T_1 \Rightarrow T_2) \Rightarrow T_2)}\ \text{T-Abs}$$

STLC type derivation tree debugging (2)

$$\dfrac{\dfrac{\text{unbound variable } a_1}{[a_4 \mapsto T_2\ ] \vdash a_1 : T_3}\ \text{T-Var}}{[] \vdash (\lambda a_0 . a_1) : (T_2 \Rightarrow T_3)}\ \text{T-Abs}$$

STLC type derivation tree debugging (3)

$$\dfrac{\text{no case for } [a_4 \mapsto T_1\ ] \vdash (a_4\, a_4\, a_4) : T_2}{[] \vdash (\lambda a_0 . (a_0\, a_0\, a_0)) : (T_1 \Rightarrow T_2)}\ \text{T-Abs}$$