The original DOT Calculus

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Syntax

x, y, z	Variable	$L ::=$	Type label
l	Value label	$\quad L_c$	class label
$v ::=$	Value	$\quad L_a$	abstract type label
$\quad x$	variable	$S, T, U, V ::=$	Type
$\quad \lambda x\!:\!T.t$	function	$\quad p.L$	type selection
$t ::=$	Term	$\quad T\left\{z \Rightarrow \overline{D}\right\}$	refinement
$\quad v$	value	$\quad T \rightarrow T$	function type
$\quad t\ t$	application	$\quad T \wedge T$	intersection type
$\quad \mathbf{val}\ x = \mathbf{new}\ c;\ t$	new instance	$\quad T \vee T$	union type
$\quad t.l$	selection	$\quad \top$	top type
$p ::=$	Path	$\quad \bot$	bottom type
$\quad x$	variable	$S_c, T_c ::=$	Concrete type
$\quad p.l$	selection	$\quad p.L_c \mid T_c\left\{z \Rightarrow \overline{D}\right\} \mid T_c \wedge T_c \mid \top$	
$c ::= T_c\left\{\overline{l = v}\right\}$	Constructor	$D ::=$	Declaration
$\Gamma ::= \overline{x : T}$	Environment	$\quad L : S..U$	type declaration
$s ::= \overline{x \mapsto c}$	Store	$\quad l : T$	value declaration

Reduction $\boxed{t \mid s \ \rightarrow\ t' \mid s'}$

$$(\lambda x\!:\!T.t\,)v \mid s \ \rightarrow\ [v/x]t \mid s \qquad (\beta_v)$$

$$\mathbf{val}\ x = \mathbf{new}\ c;\ t \mid s \ \rightarrow\ t \mid s, x \mapsto c \qquad (\textsc{new})$$

$$\frac{x \mapsto T_c\left\{\overline{l = v}\right\} \in s}{x.l_i \mid s \ \rightarrow\ v_i \mid s} \qquad (\textsc{sel})$$

$$\frac{t \mid s \ \rightarrow\ t' \mid s'}{e[t] \mid s \ \rightarrow\ e[t'] \mid s'} \qquad (\textsc{context})$$

where evaluation context $\qquad e ::= [\,] \mid e\ t \mid v\ e \mid e.l$

Type Assignment $\boxed{\Gamma \vdash t : T}$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \qquad (\textsc{var})$$

$$\frac{\Gamma \vdash t \ni l : T'}{\Gamma \vdash t.l : T'} \qquad (\textsc{sel})$$

$$\frac{\Gamma \vdash t : S \rightarrow T\ ,\ t' : T'\ ,\ T' <: S}{\Gamma \vdash t\ t' : T} \qquad (\textsc{app})$$

$$\frac{x \notin fn(T) \qquad \Gamma \vdash S\ \mathbf{wf} \qquad \Gamma, x : S \vdash t : T}{\Gamma \vdash \lambda x\!:\!S.t : S \rightarrow T} \qquad (\textsc{abs})$$

$$\frac{x \notin fn(T') \qquad \Gamma \vdash T_c\ \mathbf{wf}\ ,\ T_c \prec_x \overline{L : S..U}, \overline{l : V} \qquad \Gamma, x : T_c \vdash \overline{S <: U}\ ,\ \overline{v : V'}\ ,\ \overline{V' <: V}\ ,\ t : T'}{\Gamma \vdash \mathbf{val}\ x = \mathbf{new}\ T_c\left\{\overline{l = v}\right\};\ t : T'} \qquad (\textsc{new})$$

Fig. 1. The DOT Calculus : Syntax, Reduction, Type Assignment

Membership $\boxed{\Gamma \vdash t \ni D}$

$$\frac{\Gamma \vdash p : T \ , \ T \prec_z \overline{D}}{\Gamma \vdash p \ni [p/z]D_i} \quad (\textsc{path-}\ni)$$

$$\frac{z \notin fn(D_i) \qquad \Gamma \vdash t : T \ , \ T \prec_z \overline{D}}{\Gamma \vdash t \ni D_i} \quad (\textsc{term-}\ni)$$

Expansion $\boxed{\Gamma \vdash T \prec_z \overline{D}}$

$$\frac{\Gamma \vdash T \prec_z \overline{D'}}{\Gamma \vdash T \left\{z \Rightarrow \overline{D}\right\} \prec_z \overline{D'} \wedge \overline{D}} \quad (\textsc{rfn-}\prec)$$

$$\frac{\Gamma \vdash p \ni L : S..U \ , \ U \prec_z \overline{D}}{\Gamma \vdash p.L \prec_z \overline{D}} \quad (\textsc{tsel-}\prec)$$

$$\frac{\Gamma \vdash T_1 \prec_z \overline{D}_1 \ , \ T_2 \prec_z \overline{D}_2}{\Gamma \vdash T_1 \wedge T_2 \prec_z \overline{D}_1 \wedge \overline{D}_2} \quad (\wedge\text{-}\prec)$$

$$\frac{\Gamma \vdash T_1 \prec_z \overline{D}_1 \ , \ T_2 \prec_z \overline{D}_2}{\Gamma \vdash T_1 \vee T_2 \prec_z \overline{D}_1 \vee \overline{D}_2} \quad (\vee\text{-}\prec)$$

$$\Gamma \vdash \top \prec_z \{\} \quad (\top\text{-}\prec)$$

$$\Gamma \vdash S \rightarrow T \prec_z \{\} \quad (\rightarrow\text{-}\prec)$$

$$\Gamma \vdash \bot \prec_z \overline{D_\bot} \quad (\bot\text{-}\prec)$$

Fig. 2. The DOT Calculus : Membership and Expansion

Subtyping $\boxed{\Gamma \vdash S <: T}$

$$\Gamma \vdash T <: T \qquad (\textsc{Refl})$$

$$\frac{\Gamma \vdash T <: S \ , \ S' <: T'}{\Gamma \vdash S \to S' <: T \to T'} \qquad (<:\text{-}\to)$$

$$\frac{\Gamma \vdash S <: T \ , \ S \prec_z \overline{D'} \qquad \Gamma, z : S \vdash \overline{D' <: D}}{\Gamma \vdash S <: T\left\{z \Rightarrow \overline{D}\right\}} \qquad (<:\text{-}\textsc{Rfn})$$

$$\frac{\Gamma \vdash T <: T'}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} <: T'} \qquad (\textsc{Rfn}\text{-}<:)$$

$$\frac{\Gamma \vdash p \ni L : S..U \ , \ S <: U \ , \ S' <: S}{\Gamma \vdash S' <: p.L} \qquad (<:\text{-}\textsc{TSel})$$

$$\frac{\Gamma \vdash p \ni L : S..U \ , \ S <: U \ , \ U <: U'}{\Gamma \vdash p.L <: U'} \qquad (\textsc{TSel}\text{-}<:)$$

$$\frac{\Gamma \vdash T <: T_1 \ , \ T <: T_2}{\Gamma \vdash T <: T_1 \wedge T_2} \qquad (<:\text{-}\wedge)$$

$$\frac{\Gamma \vdash T_i <: T}{\Gamma \vdash T_1 \wedge T_2 <: T} \qquad (\wedge\text{-}<:)$$

$$\frac{\Gamma \vdash T <: T_i}{\Gamma \vdash T <: T_1 \vee T_2} \qquad (<:\text{-}\vee)$$

$$\frac{\Gamma \vdash T_1 <: T \ , \ T_2 <: T}{\Gamma \vdash T_1 \vee T_2 <: T} \qquad (\vee\text{-}<:)$$

$$\Gamma \vdash T <: \top \qquad (<:\text{-}\top)$$

$$\Gamma \vdash \bot <: T \qquad (\bot\text{-}<:)$$

Declaration subsumption $\boxed{\Gamma \vdash D <: D'}$

$$\frac{\Gamma \vdash S' <: S \ , \ T <: T'}{\Gamma \vdash (L : S..T) <: (L : S'..T')} \ (\textsc{TDecl}\text{-}<:)$$

$$\frac{\Gamma \vdash T <: T'}{\Gamma \vdash (l : T) <: (l : T')} \ (\textsc{VDecl}\text{-}<:)$$

Fig. 3. The DOT Calculus : Subtyping and Declaration Subsumption

Well-formed types $\boxed{\Gamma \vdash T \ \mathbf{wf}}$

$$\frac{\Gamma \vdash T \ \mathbf{wf} \qquad \Gamma, z : T\left\{z \Rightarrow \overline{D}\right\} \vdash \overline{D \ \mathbf{wf}}}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} \ \mathbf{wf}} \ (\text{RFN-WF})$$

$$\frac{\Gamma \vdash T \ \mathbf{wf} \ , \ T' \ \mathbf{wf}}{\Gamma \vdash T \to T' \ \mathbf{wf}} \ (\to\text{-WF})$$

$$\frac{\Gamma \vdash p \ni L : S..U \ , \ S \ \mathbf{wf} \ , \ U \ \mathbf{wf}}{\Gamma \vdash p.L \ \mathbf{wf}} \ (\text{TSEL-WF}_1)$$

$$\frac{\Gamma \vdash p \ni L : \bot..U}{\Gamma \vdash p.L \ \mathbf{wf}} \ (\text{TSEL-WF}_2)$$

$$\frac{\Gamma \vdash T \ \mathbf{wf} \ , \ T' \ \mathbf{wf}}{\Gamma \vdash T \wedge T' \ \mathbf{wf}} \ (\wedge\text{-WF})$$

$$\frac{\Gamma \vdash T \ \mathbf{wf} \ , \ T' \ \mathbf{wf}}{\Gamma \vdash T \vee T' \ \mathbf{wf}} \ (\vee\text{-WF})$$

$$\Gamma \vdash \bot \ \mathbf{wf} \ (\bot\text{-WF})$$

$$\Gamma \vdash \top \ \mathbf{wf} \ (\top\text{-WF})$$

Well-formed declarations $\boxed{\Gamma \vdash D \ \mathbf{wf}}$

$$\frac{\Gamma \vdash S \ \mathbf{wf} \ , \ U \ \mathbf{wf}}{\Gamma \vdash L : S..U \ \mathbf{wf}} \ (\text{TDECL-WF})$$

$$\frac{\Gamma \vdash T \ \mathbf{wf}}{\Gamma \vdash l : T \ \mathbf{wf}} \ (\text{VDECL-WF})$$

Fig. 4. The DOT Calculus : Well-Formedness

$$
\begin{array}{lcll}
dom(\overline{D} \wedge \overline{D'}) & = & dom(\overline{D}) \cup dom(\overline{D'}) & \\
dom(\overline{D} \vee \overline{D'}) & = & dom(\overline{D}) \cap dom(\overline{D'}) & \\
(D \wedge D')(L) & = & L : (S \vee S')..(U \wedge U') & \text{if } (L : S..U) \in \overline{D} \text{ and } (L : S'..U') \in \overline{D'} \\
 & = & D(L) & \text{if } L \notin dom(\overline{D'}) \\
 & = & D'(L) & \text{if } L \notin dom(\overline{D}) \\
(D \wedge D')(l) & = & l : T \wedge T' & \text{if } (l : T) \in \overline{D} \text{ and } (l : T') \in \overline{D'} \\
 & = & D(l) & \text{if } l \notin dom(\overline{D'}) \\
 & = & D'(l) & \text{if } l \notin dom(\overline{D}) \\
(D \vee D')(L) & = & L : (S \wedge S')..(U \vee U') & \text{if } (L : S..U) \in \overline{D} \text{ and } (L : S'..U') \in \overline{D'} \\
(D \vee D')(l) & = & l : T \vee T' & \text{if } (l : T) \in \overline{D} \text{ and } (l : T') \in \overline{D'}
\end{array}
$$

Sets of declarations form a lattice with the given meet $\wedge$ and join $\vee$, the empty set of declatations as the top element, and the bottom element $\overline{D_\bot}$, Here $\overline{D_\bot}$ is the set of declarations that contains for every term label l the declaration $l : \bot$ and for every type label L the declaration $L : \top..\bot$.

Fig. 5. The DOT Calculus : Declaration Lattice