minidot

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Expansion $\boxed{G \vdash T \prec_N T'}$

$$G \vdash \top \prec_N \emptyset_T \qquad \text{(EXP-TP/TOP)}$$

$$G \vdash \bot \prec_N \emptyset_T \qquad \text{(EXP-TP/BOT)}$$

$$G \vdash \{\textbf{def } N1 : T1 \to T2\} \prec_{N2} \emptyset_T \quad \text{(EXP-TP/FUN)}$$

$$G \vdash \{\textbf{val } N1 : T\} \prec_{N2} \emptyset_T \quad \text{(EXP-TP/RECV)}$$

$$G \vdash \{\textbf{type } N : T1 .. T2\} \prec_N \{\textbf{type } N : T1 .. T2\} \\ \text{(EXP-TP/RECT)}$$

$$\frac{N1 \neq N2}{G \vdash \{\textbf{type } N1 : T1 .. T2\} \prec_{N2} \emptyset_T} \quad \text{(EXP-TP/RECTN)}$$

$$\frac{\begin{array}{c} T1' \wedge T2' = T3' \\ G \vdash T1 \prec_N T1' \\ G \vdash T2 \prec_N T2' \end{array}}{G \vdash T1 \wedge T2 \prec_N T3'} \quad \text{(EXP-TP/AND)}$$

$$\frac{\begin{array}{c} T1' \vee T2' = T3' \\ G \vdash T1 \prec_N T1' \\ G \vdash T2 \prec_N T2' \end{array}}{G \vdash T1 \vee T2 \prec_N T3'} \quad \text{(EXP-TP/OR)}$$

$$\frac{\begin{array}{c} \{ N1 \Rightarrow T1 \}::G \vdash T1 \prec_{N2} T2 \\ G \sqsubseteq_{N1} G0 \end{array}}{G0 \vdash \{ N1 \Rightarrow T1 \} \prec_{N2} T2} \quad \text{(EXP-TP/BIND)}$$

$$\frac{\begin{array}{c} G \vdash TB \prec_N T \\ G \vdash E \ni_0 \{\textbf{type } L : TA .. TB\} \end{array}}{G \vdash E.L \prec_N T} \quad \text{(EXP-TP/TSEL)}$$

Expanding Intersections $\boxed{T1 \wedge T2 = T3}$

$$\emptyset_T \wedge \emptyset_T = \emptyset_T \qquad \text{(TOPT-AND/NN)}$$

$$\emptyset_T \wedge T = T \qquad \text{(TOPT-AND/NS)}$$

$$T \wedge \emptyset_T = T \qquad \text{(TOPT-AND/SN)}$$

$$\{\textbf{type } N : TA1 .. TB1\} \wedge \{\textbf{type } N : TA2 .. TB2\} = \{\textbf{type } N : TA1 \vee TA2 .. TB1 \wedge TB2\} \qquad \text{(TOPT-AND/SS)}$$

Expanding Unions $\boxed{T1 \vee T2 = T3}$

$$\emptyset_T \vee \emptyset_T = \emptyset_T \qquad \text{(TOPT-OR/NN)}$$

$$\emptyset_T \vee T = \emptyset_T \qquad \text{(TOPT-OR/NS)}$$

$$T \vee \emptyset_T = \emptyset_T \qquad \text{(TOPT-OR/SN)}$$

$$\{\textbf{type } N : TA1 .. TB1\} \vee \{\textbf{type } N : TA2 .. TB2\} = \{\textbf{type } N : TA1 \wedge TA2 .. TB1 \vee TB2\} \qquad \text{(TOPT-OR/SS)}$$

Expanded Lookup $\boxed{G \vdash E \ni_0 T}$

$$\frac{\begin{array}{c} G \vdash T0 \prec_{LN} \{\textbf{type } LN : T1 .. T2\} \\ N \mapsto T0 \in G \end{array}}{G \vdash \text{var}(N) \ni_0 \{\textbf{type } LN : T1 .. T2\}} \qquad \text{(TLE)}$$

Membership $\boxed{G \vdash E \ni_M T}$

$$\frac{G \vdash E \ni_0 \{\textbf{type } N : TS .. TU\}}{G \vdash E \ni_M \{\textbf{type } N : TS .. TU\}} \qquad \text{(PEV)}$$

Figure 1. Expansion and Membership

Subtyping

$$\boxed{G1 \vdash T1 <:_M T2 \dashv G2}$$

$$\frac{\begin{array}{c} G \vdash T \ \mathbf{wf_!} \\ G \sqsubseteq G2 \\ G \sqsubseteq G1 \end{array}}{G1 \vdash T <:_! T \dashv G2} \quad \text{(SUB-TP/REFL)}$$

$$\frac{G1 \vdash T \ \mathbf{wf_M}}{G1 \vdash T <:_M \top \dashv G2} \quad \text{(SUB-TP/TOP)}$$

$$\frac{\begin{array}{c} G1 \vdash TB1 <:_M TB2 \dashv G2 \\ G2 \vdash TA2 <:_M TA1 \dashv G1 \end{array}}{G1 \vdash \{\mathbf{def}\ N : TA1 \to TB1\} <:_M \{\mathbf{def}\ N : TA2 \to TB2\} \dashv G2} \quad \text{(SUB-TP/FUN)}$$

$$\frac{\begin{array}{c} G1 \vdash TB1 <:_M TB2 \dashv G2 \\ G2 \vdash TA2 <:_M TA1 \dashv G1 \\ G2 \vdash TA2 <:_M TB2 \dashv G1 \\ G1 \vdash TA1 <:_M TB1 \dashv G1 \end{array}}{G1 \vdash \{\mathbf{type}\ N : TA1 .. TB1\} <:_M \{\mathbf{type}\ N : TA2 .. TB2\} \dashv G2} \quad \text{(SUB-TP/RECT)}$$

$$\frac{G1 \vdash T1 <:_M T2 \dashv G2}{G1 \vdash \{\mathbf{val}\ N : T1\} <:_M \{\mathbf{val}\ N : T2\} \dashv G2} \quad \text{(SUB-TP/RECV)}$$

$$\frac{\begin{array}{c} G1X \vdash \{\mathbf{type}\ N : TA1 .. TB1\} <:_! \{\mathbf{type}\ N : TA2 .. TB2\} \dashv G2X \\ G2X \sqsubseteq_{s(X)} G2 \\ G1X \sqsubseteq_{s(X)} G1 \\ G2 \vdash var(X) \ni_! \{\mathbf{type}\ N : TA2 .. TB2\} \\ G1 \vdash var(X) \ni_! \{\mathbf{type}\ N : TA1 .. TB1\} \end{array}}{G1 \vdash var(X).N <:_! var(X).N \dashv G2} \quad \text{(SUB-TP/TSELX-STRICT)}$$

$$\frac{\begin{array}{c} G1 \vdash T1 <:_M TN \dashv \{ N \Rightarrow T1' \}::G0 \\ G1 \vdash T1 <:_M T1' \dashv G1' \\ G1 \vdash T1 <:_M TN \dashv \{ N \Rightarrow TN \}::G0 \\ G0 \sqsubseteq_N G2 \end{array}}{G1 \vdash T1 <:_M \{ N \Rightarrow TN \} \dashv G2} \quad \text{(SUB-TP/TBIND2)}$$

$$\frac{\begin{array}{c} \{ N \Rightarrow TN \}::G0 \vdash TN <:_M T2 \dashv G2 \\ G0 \sqsubseteq_N G1 \end{array}}{G1 \vdash \{ N \Rightarrow TN \} <:_M T2 \dashv G2} \quad \text{(SUB-TP/TBIND1)}$$

$$\frac{\begin{array}{c} G1 \vdash T1 <:_M TA2 \dashv G2 \\ G1 \vdash T1 <:_M TB2 \dashv G2 \end{array}}{G1 \vdash T1 <:_M TA2 \wedge TB2 \dashv G2} \quad \text{(SUB-TP/AND2)}$$

$$\frac{\begin{array}{c} G1 \vdash TB1 \ \mathbf{wf_M} \\ G1 \vdash TA1 <:_M T2 \dashv G2 \end{array}}{G1 \vdash TA1 \wedge TB1 <:_M T2 \dashv G2} \quad \text{(SUB-TP/AND1A)}$$

$$\frac{\begin{array}{c} G1 \vdash TA1 \ \mathbf{wf_M} \\ G1 \vdash TB1 <:_M T2 \dashv G2 \end{array}}{G1 \vdash TA1 \wedge TB1 <:_M T2 \dashv G2} \quad \text{(SUB-TP/AND1B)}$$

$$\frac{\begin{array}{c} G1 \vdash TA1 <:_M T2 \dashv G2 \\ G1 \vdash TB1 <:_M T2 \dashv G2 \end{array}}{G1 \vdash TA1 \vee TB1 <:_M T2 \dashv G2} \quad \text{(SUB-TP/OR2)}$$

$$\frac{\begin{array}{c} G2 \vdash TB2 \ \mathbf{wf_M} \\ G1 \vdash T1 <:_M TA2 \dashv G2 \end{array}}{G1 \vdash T1 <:_M TA2 \vee TB2 \dashv G2} \quad \text{(SUB-TP/OR1A)}$$

$$\frac{\begin{array}{c} G2 \vdash TA2 \ \mathbf{wf_M} \\ G1 \vdash T1 <:_M TB2 \dashv G2 \end{array}}{G1 \vdash T1 <:_M TA2 \vee TB2 \dashv G2} \quad \text{(SUB-TP/OR1B)}$$

$$\frac{\begin{array}{c} G1 \vdash var(X).N \ \mathbf{wf_M} \\ G1X \vdash TB1 <:_M T2 \dashv G2 \\ G1X \sqsubseteq_{s(X)} G1 \\ G1 \vdash var(X) \ni_M \{\mathbf{type}\ N : TA1 .. TB1\} \end{array}}{G1 \vdash var(X).N <:_M T2 \dashv G2} \quad \text{(SUB-TP/TSEL1)}$$

$$\frac{\begin{array}{c} G2 \vdash var(X).N \ \mathbf{wf_!} \\ G1 \vdash T1 <:_! TA2 \dashv G2X \\ G2X \sqsubseteq_{s(X)} G2 \\ G2 \vdash var(X) \ni_! \{\mathbf{type}\ N : TA2 .. TB2\} \end{array}}{G1 \vdash T1 <:_! var(X).N \dashv G2} \quad \text{(SUB-TP/TSEL2-STRICT)}$$

$$\frac{\begin{array}{c} G2 \vdash var(X).N \ \mathbf{wf_\approx} \\ G1 \vdash T1 <:_\approx TB2 \dashv G2X \\ G2X \sqsubseteq_{s(X)} G2 \\ G2 \vdash var(X) \ni_\approx \{\mathbf{type}\ N : TA2 .. TB2\} \end{array}}{G1 \vdash T1 <:_\approx var(X).N \dashv G2} \quad \text{(SUB-TP/TSEL2-LENIENT)}$$

$$\frac{G2 \vdash T \ \mathbf{wf_M}}{G1 \vdash \bot <:_M T \dashv G2} \quad \text{(SUB-TP/BOT)}$$

Figure 2. Subtyping

Well-Formedness $\boxed{G \vdash T\ \mathbf{wf_M}}$

$$G \vdash \top\ \mathbf{wf_M} \qquad \text{(WF-TP/TOP)}$$

$$\dfrac{GX \vdash \{\mathbf{type}\ N : TA .. TB\}\ \mathbf{wf_M} \quad GX \sqsubseteq_{s(X)} G \quad G \vdash var(X) \ni_M \{\mathbf{type}\ N : TA .. TB\}}{G \vdash var(X).N\ \mathbf{wf_M}}\ \text{(WF-TP/TSEL)}$$

$$G \vdash \bot\ \mathbf{wf_M} \qquad \text{(WF-TP/BOT)}$$

$$\dfrac{G \vdash TB\ \mathbf{wf_M} \quad G \vdash TA\ \mathbf{wf_M}}{G \vdash \{\mathbf{def}\ N : TA \to TB\}\ \mathbf{wf_M}}\ \text{(WF-TP/FUN)}$$

$$\dfrac{\{N \Rightarrow TN\}::G0 \vdash TN\ \mathbf{wf_M} \quad G0 \sqsubseteq_N G}{G \vdash \{N \Rightarrow TN\}\ \mathbf{wf_M}}\ \text{(WF-TP/TBIND)}$$

$$\dfrac{G \vdash TA <:_M TB \dashv G \quad G \vdash TB\ \mathbf{wf_M} \quad G \vdash TA\ \mathbf{wf_M}}{G \vdash \{\mathbf{type}\ N : TA .. TB\}\ \mathbf{wf_M}}\ \text{(WF-TP/RECT)}$$

$$\dfrac{G \vdash TA\ \mathbf{wf_M} \quad G \vdash TB\ \mathbf{wf_M}}{G \vdash TA \wedge TB\ \mathbf{wf_M}}\ \text{(WF-TP/AND)}$$

$$\dfrac{G \vdash T\ \mathbf{wf_M}}{G \vdash \{\mathbf{val}\ N : T\}\ \mathbf{wf_M}}\ \text{(WF-TP/RECV)}$$

$$\dfrac{G \vdash TA\ \mathbf{wf_M} \quad G \vdash TB\ \mathbf{wf_M}}{G \vdash TA \vee TB\ \mathbf{wf_M}}\ \text{(WF-TP/OR)}$$

Figure 3. Well-Formedness

Typing $\boxed{G \vdash E : T}$

$$G \vdash \{\emptyset_e\} : \top \qquad \text{(T/EMPTY)}$$

$$\dfrac{G \vdash T\ \mathbf{wf_!} \quad N \mapsto T \in G}{G \vdash var(N) : T} \qquad \text{(T/VAR)}$$

$$\dfrac{G \vdash E1 : \{\mathbf{val}\ LNV : T1\}}{G \vdash E1.LNV : T1} \qquad \text{(T/SEL)}$$

$$\dfrac{\begin{array}{c} TC::G \vdash \{\mathbf{def}\ LNF : T3 \to T4\} \wedge \{\mathbf{val}\ LNV : T2\} \wedge MT <:_! TC \dashv TC::G \\ G \vdash TC\ \mathbf{wf_!} \\ MT::G \vdash MT\ \mathbf{wf_!} \\ MT::G \vdash T2 <:_! T2 \dashv TC::G \\ MT::G \vdash R2 : T2 \\ T3::TC::G \vdash R : T4 \\ \text{type-mem}(M,MT) \\ |G|=N \end{array}}{G \vdash \mathbf{new}\ TC\ \{\mathbf{def}\ LNF(\_:T3):R=T4;\mathbf{val}\ LNV:R2=T2;\mathbf{types}\ M\} : TC} \qquad \text{(T/FUN)}$$

$$\dfrac{G \vdash E2 : T1 \quad G \vdash E1 : \{\mathbf{def}\ LNF : T1 \to T2\}}{G \vdash E1.LNF(E2) : T2} \qquad \text{(T/APP)}$$

$$\dfrac{G \vdash T1 <:_! T2 \dashv G \quad G \vdash E : T1}{G \vdash E : T2} \qquad \text{(T/SUB)}$$

Figure 4. Typing

Evaluation $\boxed{\text{H} \vdash \text{E} \Downarrow \text{V}}$

$$\text{G} \vdash \{\emptyset_e\} \Downarrow \{\emptyset_v\} \qquad \text{(E/EMPTY)}$$

$$\frac{\text{N} \mapsto \text{V} \in \text{G}}{\text{G} \vdash \text{var(N)} \Downarrow \text{V}} \qquad \text{(E/VAR)}$$

$$\frac{< \{\textbf{def } z = \{\emptyset_e\}; \textbf{val } z = \{\emptyset_v\}\} \textbf{ in } \text{G}>_v :: \text{G} \vdash \text{R2} \Downarrow \text{V2}}{\text{G} \vdash \textbf{new } \text{TC } \{\textbf{def } \text{LNF(\_:X2):R=X3}; \textbf{val } \text{LNV:R2=X4}; \textbf{types } \text{MT}\} \Downarrow < \{\textbf{def } \text{LNF = R}; \textbf{val } \text{LNV = V2}\} \textbf{ in } \text{G}>_v} \qquad \text{(E/FUN)}$$

$$\frac{\begin{array}{c} \text{V2}::< \{\textbf{def } \text{LNF = R}; \textbf{val } \text{LNV = R2}\} \textbf{ in } \text{G1}>_v :: \text{G1} \vdash \text{R} \Downarrow \text{V3} \\ \text{G} \vdash \text{E2} \Downarrow \text{V2} \\ \text{G} \vdash \text{E1} \Downarrow < \{\textbf{def } \text{LNF = R}; \textbf{val } \text{LNV = R2}\} \textbf{ in } \text{G1}>_v \end{array}}{\text{G} \vdash \text{E1.LNF(E2)} \Downarrow \text{V3}} \qquad \text{(E/APP)}$$

$$\frac{\text{G} \vdash \text{E1} \Downarrow < \{\textbf{def } \text{LNF = R}; \textbf{val } \text{LNV = V}\} \textbf{ in } \text{G1}>_v}{\text{G} \vdash \text{E1.LNV} \Downarrow \text{V}} \qquad \text{(E/SEL)}$$

Figure 5. Evaluation

Value Typing $\boxed{\text{H} \vdash \text{E } \textbf{wf}_v}$

$$\{\emptyset_v\} \vdash \text{G } \textbf{wf}_v \qquad \text{(WFV/EMPTY)}$$

$$\frac{\begin{array}{c} \text{TC0::GC} \vdash \{\textbf{def } \text{LNF} : \text{T1} \rightarrow \text{T2}\} \wedge \{\textbf{val } \text{LNV} : \text{T}\} \wedge \text{TX} <:_\approx \text{TC0} \dashv \text{TC0::GC} \\ \text{TC0::GC} \vdash \text{TC0} <:_\approx \text{TC} \dashv \text{G} \\ \text{type-mem(M,TX)} \\ \text{R2} \vdash \text{TX::GC } \textbf{wf}_v \\ \text{TX::GC} \vdash \text{T} <:_\approx \text{T} \dashv \text{TC0::GC} \\ \text{T1::TC0::GC} \vdash \text{R} : \text{T2} \\ \text{wf-env(H,GC)} \end{array}}{< \{\textbf{def } \text{LNF = R}; \textbf{val } \text{LNV = R2}\} \textbf{ in } \text{H}>_v \vdash \text{G } \textbf{wf}_v} \qquad \text{(WFV/F)}$$

$$\frac{\text{G} \vdash \{\textbf{type } \text{LNT} : \text{T1 .. T2}\} \textbf{wf}_\approx}{< \{\textbf{def } \text{LNF = R}; \textbf{val } \text{LNV = R2}\} \textbf{ in } \text{H}>_v \vdash \text{G } \textbf{wf}_v} \qquad \text{(WFV/T)}$$

$$\frac{\begin{array}{c} \text{G1} \vdash \text{T1} <:_\approx \text{T} \dashv \text{G} \\ \text{V} \vdash \text{G1 } \textbf{wf}_v \end{array}}{\text{V} \vdash \text{G } \textbf{wf}_v} \qquad \text{(WFV/SUB)}$$

Figure 6. Value Typing