The DOT Calculus

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Syntax

x, y, z	Variable
l	Value label
m	Method label
o	Store location
$v ::=$	Value
$\quad o$	location
$w ::=$	Syntactic Value
$\quad v$	value
$\quad x$	variable
$p, q ::=$	Path
$\quad w$	value
$\quad p.l$	selection
$t ::=$	Term
$\quad p$	path
$\quad \mathbf{val}\ x = \mathbf{new}\ c;\ t$	new instance
$\quad \mathbf{val}\ x = p.m(p);\ t$	method invocation
$\quad \mathbf{val}\ x = o.m \ldots\ t;\ t$	pending met. exec.
$d ::=$	Initialization
$\quad l = w$	field init.
$\quad m(x) = t$	method init.
$c ::= T_c \left\{\overline{d}\right\}$	Constructor
$\Gamma ::= \overline{x : T}$	Environment
$s ::= \overline{o \mapsto c}$	Store

$L ::=$	Type label
$\quad L_c$	class label
$\quad L_a$	abstract type label
$S, T, U, V, W ::=$	Type
$\quad p.L$	type selection
$\quad T \left\{z \Rightarrow \overline{D}\right\}$	refinement
$\quad T \wedge T$	intersection type
$\quad T \vee T$	union type
$\quad \top$	top type
$\quad \bot$	bottom type
$S_c, T_c, U_c ::=$	Concrete type
$\quad p.L_c \mid T_c \left\{z \Rightarrow \overline{D}\right\} \mid T_c \wedge T_c \mid \top$	
$D ::=$	Declaration
$\quad L_a : S..U$	abstract type decl.
$\quad L_c : \bot..U_c$	*unique* class decl.
$\quad l : T$	value declaration
$\quad m : S \rightarrow U$	method declaration

Fig. 1. The DOT Calculus : Syntax

Reduction $\boxed{t \,|\, s \;\rightarrow\; t' \,|\, s'}$ $\boxed{p \,|\, s \;\rightarrow\; p' \,|\, s}$

$$\frac{o \mapsto T_c \left\{\overline{l = v}\ \overline{m(x) = t}\right\} \in s}{o.l_i \,|\, s \;\rightarrow\; v_i \,|\, s} \qquad (\textsc{sel})$$

$$\frac{o \mapsto T_c \left\{\overline{l = v'}\ \overline{m(x) = t}\right\} \in s}{\mathbf{val}\ y = o.m_i(v);\ t' \,|\, s \;\rightarrow\; \mathbf{val}\ y = o.m_i \ldots\ [v/x_i]t_i;\ t' \,|\, s} \qquad (\textsc{mapp})$$

$$\mathbf{val}\ y = o.m_i \ldots\ v;\ t' \,|\, s \;\rightarrow\; [v/y]t' \,|\, s \qquad (\textsc{mexe})$$

$$\frac{\begin{array}{c} o \notin dom(s) \\ c = T_c \left\{\overline{d}\right\}\ \ c' = T_c \left\{[o/x]\overline{d}\right\}\ \ t' = [o/x]t \end{array}}{\mathbf{val}\ x = \mathbf{new}\ c;\ t \,|\, s \;\rightarrow\; t \,|'\, s, o \mapsto c'} \qquad (\textsc{new})$$

$$\frac{t \,|\, s \;\rightarrow\; t' \,|\, s'}{e[t] \,|\, s \;\rightarrow\; e[t'] \,|\, s'} \qquad (\textsc{context})$$

where $e ::= [\,] \mid e.l \mid \mathbf{val}\ x = e.m(p);\ t \mid \mathbf{val}\ x = v.m(e);\ t \mid \mathbf{val}\ x = v.m \ldots\ e;\ t$

Fig. 2. The DOT Calculus : Reduction

Term Typing $\boxed{\Gamma; s \vdash t :_{<:} T}$

$$\frac{\Gamma; s \vdash p : T' \ , \ T' <: T}{\Gamma; s \vdash p :_{<:} T} \ (\textsc{path})$$

$$\frac{\Gamma; s \vdash T_c \ \mathbf{wf} \ , \ T_c \prec_y \overline{L : S..U}, \overline{D} \quad \Gamma, y : T_c; s \vdash \overline{S <: U} \ , \ \overline{d :_{<:} D} \ , \ t' :_{<:} T'}{\Gamma; s \vdash \mathbf{val}\ y = \mathbf{new}\ T_c \left\{\overline{d}\right\};\ t' :_{<:} T'} \ (\textsc{new})$$

$$\frac{\Gamma; s \vdash p \ni m : S \to T \ , \ p' :_{<:} S \quad \Gamma, y : T; s \vdash t' :_{<:} T'}{\Gamma; s \vdash \mathbf{val}\ y = p.m(p');\ t' :_{<:} T'} \ (\textsc{mapp})$$

$$\frac{\Gamma; s \vdash v \ni m : S \to T \ , \ t :_{<:} T \quad \Gamma, y : T; s \vdash t' :_{<:} T'}{\Gamma; s \vdash \mathbf{val}\ y = v.m \ldots\ t;\ t' :_{<:} T'} \ (\textsc{mexe})$$

Path Typing $\boxed{\Gamma; s \vdash p : T}$

$$\frac{x : T \in \Gamma}{\Gamma; s \vdash x : T} \ (\textsc{var})$$

$$\frac{o \mapsto T_c \left\{\overline{d}\right\} \in s}{\Gamma; s \vdash o : T_c} \ (\textsc{loc})$$

$$\frac{\Gamma; s \vdash p \ni l : T'}{\Gamma; s \vdash p.l : T'} \ (\textsc{sel})$$

Initialization Typing $\boxed{\Gamma; s \vdash d :_{<:} D}$

$$\frac{\Gamma; s \vdash w :_{<:} V}{\Gamma; s \vdash (l = w) :_{<:} (l : V)} \ (\textsc{vdecl})$$

$$\frac{\Gamma; s \vdash S \ \mathbf{wf} \quad \Gamma; s, x : S \vdash t :_{<:} T}{\Gamma; s \vdash (m(x) = t) :_{<:} (m : S \to T)} \ (\textsc{mdecl})$$

Fig. 3. The DOT Calculus : Typing

Path Resolution $\boxed{s \vdash p \Downarrow o}$

$$s \vdash o \Downarrow o \qquad (\textsc{loc-}\Downarrow)$$

$$\frac{s \vdash p \Downarrow o \quad o \mapsto T_c \left\{ \overline{l = o} \; \overline{m(x) = t} \right\} \in s}{s \vdash p.l_i \Downarrow o_i} \qquad (\textsc{path-}\Downarrow)$$

Path Irresolution $\boxed{s \vdash p \Updownarrow p}$

$$s \vdash x \Updownarrow x \qquad (\textsc{var-}\Updownarrow)$$

$$\frac{s \vdash p \Updownarrow p}{s \vdash p.l \Updownarrow p.l} \qquad (\textsc{path-}\Updownarrow)$$

Type Resolution $\boxed{s \vdash p.L \Downarrow o.L}$

$$\frac{s \vdash p \Downarrow o}{s \vdash p.L \Downarrow o.L} \qquad (\textsc{tsel-}\Downarrow)$$

Optional Type Resolution $\boxed{s \vdash p.L \Updownarrow p.L}$

$$\frac{s \vdash p \Downarrow o}{s \vdash p.L \Updownarrow o.L} \qquad (\textsc{tsel-}\Downarrow\textsc{-}\Updownarrow)$$

$$\frac{s \vdash p \Updownarrow p}{s \vdash p.L \Updownarrow p.L} \qquad (\textsc{tsel-}\Updownarrow\textsc{-}\Updownarrow)$$

Fig. 4. The DOT Calculus : Path and Type Resolution

Membership $\boxed{\Gamma; s \vdash p \ni D}$

$$\frac{\Gamma; s \vdash p : T \ , \ T \prec:_z \overline{D}}{\Gamma; s \vdash p \ni [p/z]D_i} \quad (\text{PATH-}\ni)$$

Expansion $\boxed{\Gamma; s \vdash T \prec_z \overline{D}}$

$$\Gamma; s \vdash \top \prec_z \{\} \quad (\top\text{-}\prec)$$

$$\Gamma; s \vdash M/T \prec:_z \{\} \quad (\text{ANY-}\prec:)$$

$$\frac{\Gamma; s \vdash T_1 \prec_z \overline{D}_1 \ , \ M/T_2 \prec_z \overline{D}_2}{\Gamma; s \vdash T_1 \wedge T_2 \prec_z \overline{D}_1 \wedge \overline{D}_2} \quad (\wedge\text{-}\prec)$$

$$\frac{\Gamma; s \vdash T_1 \prec:_z \overline{D}_1 \ , \ M/T_2 \prec:_z \overline{D}_2}{\Gamma; s \vdash T_1 \vee T_2 \prec:_z \overline{D}_1 \vee \overline{D}_2} \quad (\vee\text{-}\prec:)$$

$$\frac{\Gamma; s \vdash T \prec_z \overline{D}}{\Gamma; s \vdash T \left\{z \Rightarrow \overline{D}\right\} \prec_z \overline{D} \wedge \overline{D}} \quad (\text{RFN-}\prec)$$

$$\Gamma; s \vdash \bot \prec_z \overline{D_\bot} \quad (\bot\text{-}\prec)$$

$$\frac{\begin{array}{c} s \vdash p.L \Updownarrow q.L \\ \Gamma; s \vdash q \ni L : S..U \\ \Gamma; s \vdash U \prec_z \overline{D} \end{array}}{\Gamma; s \vdash p.L \prec_z \overline{D}} \quad (\text{TSEL-}\prec_2)$$

Fig. 5. The DOT Calculus : Membership and Expansion

Subtyping $\boxed{\Gamma; s \vdash S <: T}$

$$\frac{\Gamma; s \vdash T \ \mathbf{wf}}{\Gamma; s \vdash T <: T} \ (\text{REFL}) \qquad \frac{\Gamma; s \vdash T \ \mathbf{wf}}{\Gamma; s \vdash T <: \top} \ (<:\text{-}\top)$$

$$\frac{\begin{array}{c}\Gamma; s \vdash p_1.L \ \mathbf{wf} \ , \ p_2.L \ \mathbf{wf} \\ s \vdash p_1.L \Updownarrow q.L \ , \ p_2.L \Updownarrow q.L\end{array}}{\Gamma; s \vdash p_1.L <: p_2.L} \ (\text{REFL-}\Updownarrow) \qquad \frac{\Gamma; s \vdash T \ \mathbf{wf}}{\Gamma; s \vdash \bot <: T} \ (\bot\text{-}<:)$$

$$\frac{\begin{array}{c}\Gamma; s \vdash T \left\{z \Rightarrow \overline{D}\right\} \ \mathbf{wf} \ , \ S <: T \ , \ S \prec:_z \overline{D'} \\ \Gamma, z : S; s \vdash \overline{D' <: D}\end{array}}{\Gamma; s \vdash S <: T \left\{z \Rightarrow \overline{D}\right\}} \ (<:\text{-RFN}) \qquad \frac{\Gamma; s \vdash T \left\{z \Rightarrow \overline{D}\right\} \ \mathbf{wf} \ , \ T <: T'}{\Gamma; s \vdash T \left\{z \Rightarrow \overline{D}\right\} <: T'} \ (\text{RFN-}<:)$$

$$\frac{\Gamma; s \vdash T <: T_1 \ , \ T <: T_2}{\Gamma; s \vdash T <: T_1 \wedge T_2} \ (<:\text{-}\wedge) \qquad \frac{\Gamma; s \vdash T_1 <: T \ , \ T_2 <: T}{\Gamma; s \vdash T_1 \vee T_2 <: T} \ (\vee\text{-}<:)$$

$$\frac{\Gamma; s \vdash T_2 \ \mathbf{wf} \ , \ T <: T_1}{\Gamma; s \vdash T <: T_1 \vee T_2} \ (<:\text{-}\vee_1) \qquad \frac{\Gamma; s \vdash T_2 \ \mathbf{wf} \ , \ T_1 <: T}{\Gamma; s \vdash T_1 \wedge T_2 <: T} \ (\wedge_1\text{-}<:)$$

$$\frac{\Gamma; s \vdash T_1 \ \mathbf{wf} \ , \ T <: T_2}{\Gamma; s \vdash T <: T_1 \vee T_2} \ (<:\text{-}\vee_2) \qquad \frac{\Gamma; s \vdash T_1 \ \mathbf{wf} \ , \ T_2 <: T}{\Gamma; s \vdash T_1 \wedge T_2 <: T} \ (\wedge_2\text{-}<:)$$

$$\frac{\Gamma; s \vdash p \ni L : S..U \ , \ S' <: S}{\Gamma; s \vdash S' <: p.L} \ (<:\text{-TSEL}) \qquad \frac{\Gamma; s \vdash p \ni L : S..U \ , \ U <: U'}{\Gamma; s \vdash p.L <: U'} \ (\text{TSEL-}<:)$$

Declaration subsumption $\boxed{\Gamma; s \vdash D <: D'}$

$$\frac{\Gamma; s \vdash S' <: S \ , \ T <: T'}{\Gamma; s \vdash (L : S..T) <: (L : S'..T')} \ (\text{TDECL-}<:) \qquad \frac{\Gamma; s \vdash S' <: S \ , \ T <: T'}{\Gamma; s \vdash (m : S \to T) <: (m : S' \to T')} \ (\text{MDECL-}<:)$$

$$\frac{\Gamma; s \vdash T <: T'}{\Gamma; s \vdash (l : T) <: (l : T')} \ (\text{VDECL-}<:)$$

Fig. 6. The DOT Calculus : Subtyping and Declaration Subsumption

Well-formed types $\boxed{\Gamma; s \vdash T \ \mathbf{wf}}$

$$\Gamma; s \vdash \top \ \mathbf{wf} \quad (\top\text{-WF})$$

$$\Gamma; s \vdash \bot \ \mathbf{wf} \quad (\bot\text{-WF})$$

$$\frac{\Gamma; s \vdash T \ \mathbf{wf} \qquad \Gamma, z : T\left\{z \Rightarrow \overline{D}\right\}; s \vdash \overline{D \ \mathbf{wf}}}{\Gamma; s \vdash T\left\{z \Rightarrow \overline{D}\right\} \ \mathbf{wf}} \ (\text{RFN-WF})$$

$$\frac{\Gamma; s \vdash p \ni L : S..U}{\Gamma; s \vdash p.L \ \mathbf{wf}} \quad (\text{TSEL-WF})$$

$$\frac{\Gamma; s \vdash T \ \mathbf{wf} \ , \ T' \ \mathbf{wf}}{\Gamma; s \vdash T \wedge T' \ \mathbf{wf}} \quad (\wedge\text{-WF})$$

$$\frac{\Gamma; s \vdash T \ \mathbf{wf} \ , \ T' \ \mathbf{wf}}{\Gamma; s \vdash T \vee T' \ \mathbf{wf}} \quad (\vee\text{-WF})$$

Well-formed declarations $\boxed{\Gamma; s \vdash D \ \mathbf{wf}}$

$$\frac{\Gamma; s \vdash S \ \mathbf{wf} \ , \ U \ \mathbf{wf}}{\Gamma; s \vdash L : S..U \ \mathbf{wf}} \ (\text{TDECL-WF})$$

$$\frac{\Gamma; s \vdash S \ \mathbf{wf} \ , \ U \ \mathbf{wf}}{\Gamma; s \vdash m : S \to U \ \mathbf{wf}} \ (\text{MDECL-WF})$$

$$\frac{\Gamma; s \vdash T \ \mathbf{wf}}{\Gamma; s \vdash l : T \ \mathbf{wf}} \quad (\text{VDECL-WF})$$

Fig. 7. The DOT Calculus : Well-Formedness

$$
\begin{array}{lcll}
dom(\overline{D} \wedge \overline{D'}) & = & dom(\overline{D}) \cup dom(\overline{D'}) & \\
dom(\overline{D} \vee \overline{D'}) & = & dom(\overline{D}) \cap dom(\overline{D'}) & \\
(D \wedge D')(L) & = & L : (S \vee S')..(U \wedge U') & \text{if } (L : S..U) \in \overline{D} \text{ and } (L : S'..U') \in \overline{D'} \\
& = & D(L) & \text{if } L \notin dom(\overline{D'}) \\
& = & D'(L) & \text{if } L \notin dom(\overline{D}) \\
(D \wedge D')(m) & = & m : (S \vee S') \to (U \wedge U') & \text{if } (m : S \to U) \in \overline{D} \text{ and } (m : S' \to U') \in \overline{D'} \\
& = & D(m) & \text{if } m \notin dom(\overline{D'}) \\
& = & D'(m) & \text{if } m \notin dom(\overline{D}) \\
(D \wedge D')(l) & = & l : T \wedge T' & \text{if } (l : T) \in \overline{D} \text{ and } (l : T') \in \overline{D'} \\
& = & D(l) & \text{if } l \notin dom(\overline{D'}) \\
& = & D'(l) & \text{if } l \notin dom(\overline{D}) \\
(D \vee D')(L) & = & L : (S \wedge S')..(U \vee U') & \text{if } (L : S..U) \in \overline{D} \text{ and } (L : S'..U') \in \overline{D'} \\
(D \vee D')(m) & = & m : (S \wedge S') \to (U \vee U') & \text{if } (m : S \to U) \in \overline{D} \text{ and } (m : S' \to U') \in \overline{D'} \\
(D \vee D')(l) & = & l : T \vee T' & \text{if } (l : T) \in \overline{D} \text{ and } (l : T') \in \overline{D'}
\end{array}
$$

Sets of declarations form a lattice with the given meet $\wedge$ and join $\vee$, the empty set of declatations as the top element, and the bottom element $\overline{D_\bot}$, Here $\overline{D_\bot}$ is the set of declarations that contains for every term label l the declaration $l : \bot$, for every type label L the declaration $L : \top..\bot$ and for every method label m the declaration $m : \top \to \bot$.

Fig. 8. The DOT Calculus : Declaration Lattice