The DOT Calculus

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Syntax

x, y, z	Variable
l	Value label
m	Method label
$v ::=$	Value
$\quad x$	$\quad$ variable
$t ::=$	Term
$\quad v$	$\quad$ value
$\quad \textbf{val}\ x = \textbf{new}\ c;\ t$	$\quad$ new instance
$\quad t.l$	$\quad$ field selection
$\quad t.m(t)$	$\quad$ method invocation
$p ::=$	Path
$\quad x$	$\quad$ variable
$\quad p.l$	$\quad$ selection
$c ::= T_c \left\{ \overline{l = v}\ \overline{m(x) = t} \right\}$	Constructor
$\Gamma ::= \overline{x : T}$	Environment
$s ::= \overline{x \mapsto c}$	Store

$L ::=$	Type label
$\quad L_c$	$\quad$ class label
$\quad L_a$	$\quad$ abstract type label
$S, T, U, V, W ::=$	Type
$\quad p.L$	$\quad$ type selection
$\quad T \left\{ z \Rightarrow \overline{D} \right\}$	$\quad$ refinement
$\quad T \wedge T$	$\quad$ intersection type
$\quad T \vee T$	$\quad$ union type
$\quad \top$	$\quad$ top type
$\quad \bot$	$\quad$ bottom type
$S_c, T_c ::=$	Concrete type
$\quad p.L_c \mid T_c \left\{ z \Rightarrow \overline{D} \right\} \mid T_c \wedge T_c \mid \top$	
$D ::=$	Declaration
$\quad L : S..U$	$\quad$ type declaration
$\quad l : T$	$\quad$ value declaration
$\quad m : S \to U$	$\quad$ method declaration

Reduction $\boxed{t \mid s \ \to\ t' \mid s'}$

$$\frac{y \mapsto T_c \left\{ \overline{l = v'}\ \overline{m(x) = t} \right\} \in s}{y.m_i(v) \mid s \ \to\ [v/x_i]t_i \mid s} \ (\textsc{msel}) \qquad \textbf{val}\ x = \textbf{new}\ c;\ t \mid s \ \to\ t \mid s, x \mapsto c \ (\textsc{new})$$

$$\frac{y \mapsto T_c \left\{ \overline{l = v}\ \overline{m(x) = t} \right\} \in s}{y.l_i \mid s \ \to\ v_i \mid s} \ (\textsc{sel}) \qquad \frac{t \mid s \ \to\ t' \mid s'}{e[t] \mid s \ \to\ e[t'] \mid s'} \ (\textsc{context})$$

where evaluation context $\qquad e ::= [\,] \mid e.m(t) \mid v.m(e) \mid e.l$

Type Assignment $\boxed{\Gamma \vdash t : T}$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \ (\textsc{var}) \qquad \frac{\Gamma \vdash t \ni l : T'}{\Gamma \vdash t.l : T'} \ (\textsc{sel})$$

$$\frac{\begin{array}{c}\Gamma \vdash t \ni m : S \to T \\ \Gamma \vdash t' : T' \ ,\ T' <: S\end{array}}{\Gamma \vdash t.m(t') : T} \ (\textsc{msel}) \qquad \frac{\begin{array}{c} y \notin fn(T') \\ \Gamma \vdash T_c\ \textbf{wfe}\ ,\ T_c \prec_y \overline{L : S..U}, \overline{D} \\ \Gamma, y : T_c \vdash \overline{S <: U}\ ,\ \overline{d : D}\ ,\ t' : T' \end{array}}{\Gamma \vdash \textbf{val}\ y = \textbf{new}\ T_c \left\{ \overline{d} \right\};\ t' : T'} \ (\textsc{new})$$

Declaration Assignment $\boxed{\Gamma \vdash d : D}$

$$\frac{\Gamma \vdash v : V' \ ,\ V' <: V}{\Gamma \vdash (l = v) : (l : V)} \ (\textsc{vdecl}) \qquad \frac{\begin{array}{c}\Gamma \vdash S\ \textbf{wfe} \\ \Gamma, x : S \vdash t : T' \ ,\ T' <: T\end{array}}{\Gamma \vdash (m(x) = t) : (m : S \to T)} \ (\textsc{mdecl})$$

Fig. 1. The DOT Calculus : Syntax, Reduction, Type / Declaration Assignment

Membership $\boxed{\Gamma \vdash t \ni D}$

$$\frac{\Gamma \vdash p : T \;,\; T \prec_z \overline{D}}{\Gamma \vdash p \ni [p/z]D_i} \quad (\textsc{path-}\ni) \qquad \frac{z \notin fn(D_i) \qquad \Gamma \vdash t : T \;,\; T \prec_z \overline{D}}{\Gamma \vdash t \ni D_i} \quad (\textsc{term-}\ni)$$

Expansion $\boxed{\Gamma \vdash T \prec_z \overline{D}}$

$$\frac{\Gamma \vdash T \prec_z \overline{D'}}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} \prec_z \overline{D'} \wedge \overline{D}} \quad (\textsc{rfn-}\prec) \qquad \frac{\Gamma \vdash p \ni L : S..U \;,\; U \prec_z \overline{D}}{\Gamma \vdash p.L \prec_z \overline{D}} \quad (\textsc{tsel-}\prec)$$

$$\frac{\Gamma \vdash T_1 \prec_z \overline{D}_1 \;,\; T_2 \prec_z \overline{D}_2}{\Gamma \vdash T_1 \wedge T_2 \prec_z \overline{D}_1 \wedge \overline{D}_2} \quad (\wedge\text{-}\prec) \qquad \frac{\Gamma \vdash T_1 \prec_z \overline{D}_1 \;,\; T_2 \prec_z \overline{D}_2}{\Gamma \vdash T_1 \vee T_2 \prec_z \overline{D}_1 \vee \overline{D}_2} \quad (\vee\text{-}\prec)$$

$$\Gamma \vdash \top \prec_z \{\} \quad (\top\text{-}\prec) \qquad \Gamma \vdash \bot \prec_z \overline{D_\bot} \quad (\bot\text{-}\prec)$$

Fig. 2. The DOT Calculus : Membership and Expansion

Subtyping $\boxed{\Gamma \vdash S <: T}$

$$\frac{\Gamma \vdash T \ \mathbf{wfe}}{\Gamma \vdash T <: T} \quad (\textsc{refl})$$

$$\frac{\Gamma \vdash T \ \mathbf{wfe}}{\Gamma \vdash T <: \top} \quad (<:\text{-}\top)$$

$$\frac{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} \ \mathbf{wfe}\ ,\ S <: T\ ,\ S \prec_z \overline{D'} \quad \Gamma, z : S \vdash \overline{D'} <: \overline{D}}{\Gamma \vdash S <: T\left\{z \Rightarrow \overline{D}\right\}} \quad (<:\text{-}\textsc{rfn})$$

$$\frac{\Gamma \vdash T \ \mathbf{wfe}}{\Gamma \vdash \bot <: T} \quad (\bot\text{-}<:)$$

$$\frac{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} \ \mathbf{wfe}\ ,\ T <: T'}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\} <: T'} \quad (\textsc{rfn}\text{-}<:)$$

$$\frac{\Gamma \vdash p \ni L : S..U\ ,\ S <: U\ ,\ S' <: S}{\Gamma \vdash S' <: p.L} \quad (<:\text{-}\textsc{tsel})$$

$$\frac{\Gamma \vdash p \ni L : S..U\ ,\ S <: U\ ,\ U <: U'}{\Gamma \vdash p.L <: U'} \quad (\textsc{tsel}\text{-}<:)$$

$$\frac{\Gamma \vdash T <: T_1\ ,\ T <: T_2}{\Gamma \vdash T <: T_1 \wedge T_2} \quad (<:\text{-}\wedge)$$

$$\frac{\Gamma \vdash T_1 <: T\ ,\ T_2 <: T}{\Gamma \vdash T_1 \vee T_2 <: T} \quad (\vee\text{-}<:)$$

$$\frac{\Gamma \vdash T_2 \ \mathbf{wfe}\ ,\ T <: T_1}{\Gamma \vdash T <: T_1 \vee T_2} \quad (<:\text{-}\vee_1)$$

$$\frac{\Gamma \vdash T_2 \ \mathbf{wfe}\ ,\ T_1 <: T}{\Gamma \vdash T_1 \wedge T_2 <: T} \quad (\wedge_1\text{-}<:)$$

$$\frac{\Gamma \vdash T_1 \ \mathbf{wfe}\ ,\ T <: T_2}{\Gamma \vdash T <: T_1 \vee T_2} \quad (<:\text{-}\vee_2)$$

$$\frac{\Gamma \vdash T_1 \ \mathbf{wfe}\ ,\ T_2 <: T}{\Gamma \vdash T_1 \wedge T_2 <: T} \quad (\wedge_2\text{-}<:)$$

Declaration subsumption $\boxed{\Gamma \vdash D <: D'}$

$$\frac{\Gamma \vdash S' <: S\ ,\ T <: T'}{\Gamma \vdash (L : S..T) <: (L : S'..T')} \quad (\textsc{tdecl}\text{-}<:)$$

$$\frac{\Gamma \vdash S' <: S\ ,\ T <: T'}{\Gamma \vdash (m : S \to T) <: (m : S' \to T')} \quad (\textsc{mdecl}\text{-}<:)$$

$$\frac{\Gamma \vdash T <: T'}{\Gamma \vdash (l : T) <: (l : T')} \quad (\textsc{vdecl}\text{-}<:)$$

Fig. 3. The DOT Calculus : Subtyping and Declaration Subsumption

Well-formed types $\boxed{\Gamma \vdash T\ \mathbf{wf}}$

$$\frac{\Gamma \vdash T\ \mathbf{wf} \quad \Gamma, z : T\left\{z \Rightarrow \overline{D}\right\} \vdash \overline{D\ \mathbf{wf}}}{\Gamma \vdash T\left\{z \Rightarrow \overline{D}\right\}\ \mathbf{wf}}\ (\text{RFN-WF})$$

$$\Gamma \vdash \top\ \mathbf{wf} \quad (\top\text{-WF})$$

$$\Gamma \vdash \bot\ \mathbf{wf} \quad (\bot\text{-WF})$$

$$\frac{\Gamma \vdash p \ni L : S..U\ ,\ S\ \mathbf{wf}\ ,\ U\ \mathbf{wf}}{\Gamma \vdash p.L\ \mathbf{wf}}\ (\text{TSEL-WF}_1)$$

$$\frac{\Gamma \vdash p \ni L : \bot..U}{\Gamma \vdash p.L\ \mathbf{wf}}\ (\text{TSEL-WF}_2)$$

$$\frac{\Gamma \vdash T\ \mathbf{wf}\ ,\ T'\ \mathbf{wf}}{\Gamma \vdash T \wedge T'\ \mathbf{wf}}\ (\wedge\text{-WF})$$

$$\frac{\Gamma \vdash T\ \mathbf{wf}\ ,\ T'\ \mathbf{wf}}{\Gamma \vdash T \vee T'\ \mathbf{wf}}\ (\vee\text{-WF})$$

Well-formed declarations $\boxed{\Gamma \vdash D\ \mathbf{wf}}$

$$\frac{\Gamma \vdash S\ \mathbf{wf}\ ,\ U\ \mathbf{wf}}{\Gamma \vdash L : S..U\ \mathbf{wf}}\ (\text{TDECL-WF})$$

$$\frac{\Gamma \vdash S\ \mathbf{wf}\ ,\ U\ \mathbf{wf}}{\Gamma \vdash m : S \rightarrow U\ \mathbf{wf}}\ (\text{MDECL-WF})$$

$$\frac{\Gamma \vdash T\ \mathbf{wf}}{\Gamma \vdash l : T\ \mathbf{wf}}\ (\text{VDECL-WF})$$

Well-formed and expanding types $\boxed{\Gamma \vdash T\ \mathbf{wfe}}$

$$\frac{\Gamma \vdash T\ \mathbf{wf}\ ,\ T \prec_z \overline{D}}{\Gamma \vdash T\ \mathbf{wfe}}\ (\text{WFE})$$

Fig. 4. The DOT Calculus : Well-Formedness

$$
\begin{array}{lcll}
dom(\overline{D} \wedge \overline{D'}) & = & dom(\overline{D}) \cup dom(\overline{D'}) & \\
dom(\overline{D} \vee \overline{D'}) & = & dom(\overline{D}) \cap dom(\overline{D'}) & \\
(D \wedge D')(L) & = & L : (S \vee S')..(U \wedge U') & \text{if } (L : S..U) \in \overline{D} \text{ and } (L : S'..U') \in \overline{D'} \\
 & = & D(L) & \text{if } L \notin dom(\overline{D'}) \\
 & = & D'(L) & \text{if } L \notin dom(\overline{D}) \\
(D \wedge D')(m) & = & m : (S \vee S') \to (U \wedge U') & \text{if } (m : S \to U) \in \overline{D} \text{ and } (m : S' \to U') \in \overline{D'} \\
 & = & D(m) & \text{if } m \notin dom(\overline{D'}) \\
 & = & D'(m) & \text{if } m \notin dom(\overline{D}) \\
(D \wedge D')(l) & = & l : T \wedge T' & \text{if } (l : T) \in \overline{D} \text{ and } (l : T') \in \overline{D'} \\
 & = & D(l) & \text{if } l \notin dom(\overline{D'}) \\
 & = & D'(l) & \text{if } l \notin dom(\overline{D}) \\
(D \vee D')(L) & = & L : (S \wedge S')..(U \vee U') & \text{if } (L : S..U) \in \overline{D} \text{ and } (L : S'..U') \in \overline{D'} \\
(D \vee D')(m) & = & m : (S \wedge S') \to (U \vee U') & \text{if } (m : S \to U) \in \overline{D} \text{ and } (m : S' \to U') \in \overline{D'} \\
(D \vee D')(l) & = & l : T \vee T' & \text{if } (l : T) \in \overline{D} \text{ and } (l : T') \in \overline{D'}
\end{array}
$$

Sets of declarations form a lattice with the given meet $\wedge$ and join $\vee$, the empty set of declatations as the top element, and the bottom element $\overline{D_\bot}$, Here $\overline{D_\bot}$ is the set of declarations that contains for every term label l the declaration $l : \bot$, for every type label L the declaration $L : \top..\bot$ and for every method label m the declaration $m : \top \to \bot$.

Fig. 5. The DOT Calculus : Declaration Lattice