$var,\ x,\ y,\ z$		Variable		
$label$		Label		
$index,\ i,\ j,\ k,\ n$				
$vlabel,\ l$	::=			Value label
	|	$label$		
$mlabel,\ m$	::=			Method Label
	|	$label$		
$clabel,\ L_c$	::=			class label
	|	$label_c$		
$alabel,\ L_a$	::=			abstract type label
	|	$label_a$		
v	::=			Value
	|	x		variable
	|	$v : T$		widening
	|	(v)	S	
$tm,\ t$	::=			Term
	|	x		value
	|	$t : T$		widening
	|	$\mathbf{val}\, x = \mathbf{new}\, c; t$	bind x in c bind x in t	new instance
	|	$t.l$		value selection
	|	$t\ m\ t'$		method invocation
	|	(t)	S	
	|	$[t/x]t'$	M	
	|	$e[t]$	M	
	|	$\mathbf{fun}\,(x : T)\,T'\ t$		
	|	$\mathbf{app}\ t\ t'$		
$path,\ p$	::=			Path
	|	x		
	|	$p : T$		
	|	$p.l$		
	|	(p)	S	
$constr,\ c$	::=			Constructor
	|	$T_c\{\ \overline{def_i}^{\,i\in 0..n}\ \}$		
def	::=			Definition
	|	$l = v$		
	|	$m(x) = t$	bind x in t	
	|	$[t/x]def$	M	
$gamma,\ \Gamma$	::=			Environment
	|	$[]$		
	|	$\Gamma, x : T$		

store, s	::=			Store
	|	$[]$		
	|	$s, x \mapsto c$		
tlabel, L	::=			Type label
	|	$label_c$		
	|	$label_a$		
tp, T, S, U, V, W	::=			Type
	|	$p.L$		type selection
	|	$T\{z \Rightarrow \overline{D}\}$	bind z in $\overline{D}$	refinement
	|	$T \wedge T'$		intersection type
	|	$T \vee T'$		union type
	|	$\top$		top type
	|	$\bot$		bottom type
	|	(T)	S	
	|	$[t/x]T$	M	
tpc, T_c, S_c	::=			Concrete Type
	|	$p.L_c$		
	|	$T_c\{z \Rightarrow \overline{D}\}$		
	|	$T_c \wedge T_c'$		
	|	$\top$		
	|	(T_c)	S	
	|	$[t/x]T_c$	M	
decl, D	::=			Declaration
	|	$L : S..U$		type declaration
	|	$l : T$		value declaration
	|	$m : S \to T$		method declaration
	|	$[t/x]D$	M	
decls, $\overline{D}$	::=			
	|	$\{\}$	S	
	|	$\overline{D_i}^{\,i}$		
	|	$\overline{D} \wedge \overline{D}'$	M	
	|	$\overline{D} \vee \overline{D}'$	M	
	|	$\overline{D_\bot}$	M	
ctx, e	::=			Context
	|	$[\,].l$		
	|	$[\,]m\ t$		
	|	$v\ m[\,]$		
	|	$[\,] : T$		
terminals	::=			
	|	$\mapsto$		

	$\rightarrow$
	$\longrightarrow$
	$\longrightarrow_p$
	$\Rightarrow$
	$\vee$
	$\wedge$
	$\top$
	$\bot$
	$[\,]$
	$\in$
	$\ni$
	$\vdash$
	$\models$
	fn
	$\notin$
	$\equiv$
	$\downarrow$
	$\uparrow_v$
	$\uparrow_m$
	λ
	$\mid$

formula ::=

	judgement
	def **is** *def′*
	$x \mapsto c \in s$
	$x : T \in \Gamma$
	$x \notin \mathit{fn}(T)$
	$\mathit{formula}_1 \quad .. \quad \mathit{formula}_n$

Job ::=

	$v \downarrow x$	Remove Widening
	$s \vdash v.l \mid v' \uparrow_v v''$	Propagate Widening for Value Selections
	$s \vdash v.m(v') \mid \lambda x'.t' \uparrow_m t''$	Propagate Widening for Method Invocations
	$t \mid s \longrightarrow t' \mid s'$	Reduction
	$\Gamma, s \vdash p \longrightarrow_p p'$	Path Reduction
	$\Gamma, s \vdash T \equiv T'$	Type Equality
	$\Gamma, s \vdash t : T$	Type Assignment
	$\Gamma, s \vdash t \ni D$	Membership
	$\Gamma, s \vdash T \prec_z \overline{D}$	Expansion
	$\Gamma, s \vdash S <: T$	Subtyping
	$\Gamma, s \vdash D <: D'$	Declaration subsumption
	$\Gamma, s \vdash \overline{D} <: \overline{D}'$	
	$\Gamma, s \vdash T\,\mathbf{wf}$	Well-formed types
	$\Gamma, s \vdash D\,\mathbf{wf}$	Well-formed declarations
	$\Gamma, s \vdash \overline{D}\,\mathbf{wf}$	

	$\Gamma, s \vdash T\ \mathbf{wfe}$	Well-formed and expanding types
	$\Gamma \models s$	Consistent Context

judgement	::=	
	|	*Job*

user_syntax	::=	
	|	*var*
	|	*label*
	|	*index*
	|	*vlabel*
	|	*mlabel*
	|	*clabel*
	|	*alabel*
	|	*v*
	|	*tm*
	|	*path*
	|	*constr*
	|	*def*
	|	*gamma*
	|	*store*
	|	*tlabel*
	|	*tp*
	|	*tpc*
	|	*decl*
	|	*decls*
	|	*ctx*
	|	*terminals*
	|	*formula*

$\boxed{v \downarrow x}$ Remove Widening

$$\frac{}{x \downarrow x}\ \text{NOWID\_VAR}$$

$$\frac{v \downarrow x}{v : T \downarrow x}\ \text{NOWID\_WID}$$

$\boxed{s \vdash v.l \,|\, v' \uparrow_v v''}$ Propagate Widening for Value Selections

$$\frac{}{s \vdash x.l \,|\, v' \uparrow_v v'}\ \text{UP\_VALUE\_VAR}$$

$$\frac{[], s \vdash (v : T) \ni l : T'}{s \vdash (v : T).l \,|\, v' \uparrow_v (v' : T')}\ \text{UP\_VALUE\_WID}$$

$\boxed{s \vdash v.m(v') \,|\, \lambda x'.t' \uparrow_m t''}$ Propagate Widening for Method Invocations

$$\frac{}{s \vdash x.m(v') \,|\, \lambda x'.t' \uparrow_m ([v'/x']t')}\ \text{UP\_METHOD\_VAR}$$

$$\frac{[], s \vdash (v : T) \ni m : S' \to T'}{s \vdash (v : T).m(v') \,|\, \lambda x'.t' \uparrow_m (([(v' : S')/x']t') : T')}\ \text{UP\_METHOD\_WID}$$

$\boxed{t \,|\, s \longrightarrow t' \,|\, s'}$ Reduction

$$\frac{}{\mathbf{val}\, x = \mathbf{new}\, c; t \mid s \;\longrightarrow\; t \mid s, x \mapsto c}\ \text{RED\_NEW}$$

$$\frac{\begin{array}{l} x \mapsto T_c\{\, \overline{def_i}^{\,i} \,\} \in s \\ def_i \,\mathbf{is}\, l = v' \\ v \downarrow x \\ s \vdash v.l \mid v' \uparrow_v v'' \end{array}}{v.l \mid s \;\longrightarrow\; v'' \mid s}\ \text{RED\_VSEL}$$

$$\frac{\begin{array}{l} x \mapsto T_c\{\, \overline{def_i}^{\,i} \,\} \in s \\ def_i \,\mathbf{is}\, m(x) = t' \\ v \downarrow x \\ s \vdash v.m(v') \mid \lambda x.t' \uparrow_m t'' \end{array}}{v\ m\ v' \mid s \;\longrightarrow\; t'' \mid s}\ \text{RED\_MSEL}$$

$$\frac{t \mid s \;\longrightarrow\; t' \mid s'}{e[t] \mid s \;\longrightarrow\; e[t'] \mid s'}\ \text{RED\_CTX}$$

$\boxed{\Gamma, s \vdash p \;\longrightarrow_p\; p'}$ Path Reduction

$$\frac{\begin{array}{l} x \mapsto T_c\{\, \overline{def_i}^{\,i} \,\} \in s \\ def_i \,\mathbf{is}\, l = v' \\ v \downarrow x \\ v' \downarrow x' \end{array}}{\Gamma, s \vdash v.l \;\longrightarrow_p\; x'}\ \text{PATH\_RED\_BASE}$$

$$\frac{}{\Gamma, s \vdash p : T \;\longrightarrow_p\; p}\ \text{PATH\_RED\_WID}$$

$$\frac{\Gamma, s \vdash p \;\longrightarrow_p\; p'}{\Gamma, s \vdash p.l \;\longrightarrow_p\; p'.l}\ \text{PATH\_RED\_REC}$$

$\boxed{\Gamma, s \vdash T \equiv T'}$ Type Equality

$$\frac{\begin{array}{l} \Gamma, s \vdash T <: T' \\ \Gamma, s \vdash T' <: T \end{array}}{\Gamma, s \vdash T \equiv T'}\ \text{TP\_EQ\_ANY}$$

$\boxed{\Gamma, s \vdash t : T}$ Type Assignment

$$\frac{x : T \in \Gamma}{\Gamma, s \vdash x : T}\ \text{TYP\_VAR}$$

$$\frac{\Gamma, s \vdash t \ni l : T}{\Gamma, s \vdash t.l : T}\ \text{TYP\_VSEL}$$

$$\frac{\begin{array}{l} \Gamma, s \vdash t_1 \ni m : S_1 \to T_1 \\ \Gamma, s \vdash t_2 : T_2 \\ \Gamma, s \vdash T_2 \equiv S_1 \end{array}}{\Gamma, s \vdash t_1\, m\, t_2 : T_1}\ \text{TYP\_MSEL}$$

$$\frac{\begin{array}{l} \Gamma, s \vdash t : T' \\ \Gamma, s \vdash T' <: T \end{array}}{\Gamma, s \vdash t : T : T}\ \text{TYP\_WID}$$

$$\frac{\begin{array}{l} x \notin \mathit{fn}(T) \\ \Gamma, s \vdash T_c\,\mathbf{wfe} \\ \Gamma, s \vdash T_c \prec_x \overline{L_j : S_j..U_j}^{\,j}\,,\ \overline{l_k : T_k}^{\,k}\,,\ \overline{m_i : V_i \to W_i}^{\,i} \\ \Gamma, x : T_c, s \vdash t : T \\ \overline{\Gamma, x : T_c, s \vdash S_j <: U_j}^{\,j} \\ \overline{\Gamma, x : T_c, s \vdash v'_k : T'_k}^{\,k} \\ \overline{\Gamma, x : T_c, s \vdash T'_k \equiv T_k}^{\,k} \\ \overline{\Gamma, x : T_c, s \vdash V_i\,\mathbf{wfe}}^{\,i} \\ \overline{\Gamma, x : T_c, x_i : V_i, s \vdash t_i : W'_i}^{\,i} \\ \overline{\Gamma, x : T_c, s \vdash W'_i \equiv W_i}^{\,i} \end{array}}{\Gamma, s \vdash \mathbf{val}\,x = \mathbf{new}\,T_c\{\,\overline{l_k = v_k}^{\,k}\,,\ \overline{m_i(x_i) = t_i}^{\,i}\,\};t : T}\ \textsc{typ\_new}$$

$\boxed{\Gamma, s \vdash t \ni D}$ Membership

$$\frac{\begin{array}{l} \Gamma, s \vdash p : T \\ \Gamma, s \vdash T \prec_z \overline{D_i}^{\,i} \end{array}}{\Gamma, s \vdash p \ni [p/z]D_i}\ \textsc{mem\_path}$$

$$\frac{\begin{array}{l} z \notin \mathit{fn}(T) \\ \Gamma, s \vdash t : T \\ \Gamma, s \vdash T \prec_z \overline{D_i}^{\,i} \end{array}}{\Gamma, s \vdash t \ni D_i}\ \textsc{mem\_term}$$

$\boxed{\Gamma, s \vdash T \prec_z \overline{D}}$ Expansion

$$\frac{\Gamma, s \vdash T \prec_z \overline{D}'}{\Gamma, s \vdash T\{z \Rightarrow \overline{D}\} \prec_z \overline{D} \wedge \overline{D}'}\ \textsc{exp\_rfn}$$

$$\frac{\begin{array}{l} \Gamma, s \vdash T_1 \prec_z \overline{D}_1 \\ \Gamma, s \vdash T_2 \prec_z \overline{D}_2 \end{array}}{\Gamma, s \vdash T_1 \wedge T_2 \prec_z \overline{D}_1 \wedge \overline{D}_2}\ \textsc{exp\_and}$$

$$\frac{\begin{array}{l} \Gamma, s \vdash T_1 \prec_z \overline{D}_1 \\ \Gamma, s \vdash T_2 \prec_z \overline{D}_2 \end{array}}{\Gamma, s \vdash T_1 \vee T_2 \prec_z \overline{D}_1 \vee \overline{D}_2}\ \textsc{exp\_or}$$

$$\frac{\begin{array}{l} \Gamma, s \vdash p \ni L : S..U \\ \Gamma, s \vdash U \prec_z \overline{D} \end{array}}{\Gamma, s \vdash p.L \prec_z \overline{D}}\ \textsc{exp\_sel}$$

$$\frac{}{\Gamma, s \vdash \top \prec_z \{\}}\ \textsc{exp\_top}$$

$$\frac{}{\Gamma, s \vdash \bot \prec_z \overline{D_\bot}}\ \textsc{exp\_bot}$$

$\boxed{\Gamma, s \vdash S <: T}$ Subtyping

$$\frac{}{\Gamma, s \vdash T <: T}\ \textsc{sub\_refl}$$

$$\frac{\begin{array}{l} \Gamma, s \vdash S <: T \\ \Gamma, s \vdash S \prec_z \overline{D}' \\ \Gamma, z : S, s \vdash \overline{D}' <: \overline{D} \end{array}}{\Gamma, s \vdash S <: T\{z \Rightarrow \overline{D}\}}\ \textsc{sub\_rfn\_r}$$

$$\frac{\Gamma, s \vdash\ T <: T'}{\Gamma, s \vdash\ T\{z \Rightarrow \overline{D}\} <: T'}\ \text{SUB\_RFN\_L}$$

$$\frac{\begin{array}{l}\Gamma, s \vdash\ p\ \ni\ L : S..U \\ \Gamma, s \vdash\ S <: U \\ \Gamma, s \vdash\ S' <: S\end{array}}{\Gamma, s \vdash\ S' <: p.L}\ \text{SUB\_TSEL\_R}$$

$$\frac{\begin{array}{l}\Gamma, s \vdash\ p\ \ni\ L : S..U \\ \Gamma, s \vdash\ S <: U \\ \Gamma, s \vdash\ U <: U'\end{array}}{\Gamma, s \vdash\ p.L <: U'}\ \text{SUB\_TSEL\_L}$$

$$\frac{\begin{array}{c}\Gamma, s \vdash\ T <: T_1 \\ \Gamma, s \vdash\ T <: T_2\end{array}}{\Gamma, s \vdash\ T <: T_1 \wedge T_2}\ \text{SUB\_AND\_R}$$

$$\frac{\Gamma, s \vdash\ T_i <: T}{\Gamma, s \vdash\ T_1 \wedge T_2 <: T}\ \text{SUB\_AND\_L}$$

$$\frac{\Gamma, s \vdash\ T <: T_i}{\Gamma, s \vdash\ T <: T_1 \vee T_2}\ \text{SUB\_OR\_R}$$

$$\frac{\begin{array}{c}\Gamma, s \vdash\ T_1 <: T \\ \Gamma, s \vdash\ T_2 <: T\end{array}}{\Gamma, s \vdash\ T_1 \vee T_2 <: T}\ \text{SUB\_OR\_L}$$

$$\frac{}{\Gamma, s \vdash\ T <: \top}\ \text{SUB\_TOP}$$

$$\frac{}{\Gamma, s \vdash\ \bot <: T}\ \text{SUB\_BOT}$$

$$\frac{\begin{array}{l}\Gamma, s \vdash\ p_1\ \longrightarrow_p\ p_2 \\ \Gamma, s \vdash\ T <: p_2.L\end{array}}{\Gamma, s \vdash\ T <: p_1.L}\ \text{SUB\_PATH\_RED}$$

$\boxed{\Gamma, s \vdash\ D <: D'}$ Declaration subsumption

$$\frac{\begin{array}{c}\Gamma, s \vdash\ S' <: S \\ \Gamma, s \vdash\ U <: U'\end{array}}{\Gamma, s \vdash\ L : S..U <: L : S'..U'}\ \text{DECL\_SUB\_TYPE}$$

$$\frac{\Gamma, s \vdash\ T <: T'}{\Gamma, s \vdash\ l : T <: l : T'}\ \text{DECL\_SUB\_VALUE}$$

$$\frac{\begin{array}{c}\Gamma, s \vdash\ S' <: S \\ \Gamma, s \vdash\ T <: T'\end{array}}{\Gamma, s \vdash\ m : S \to T <: m : S' \to T'}\ \text{DECL\_SUB\_METHOD}$$

$\boxed{\Gamma, s \vdash\ \overline{D} <: \overline{D}'}$

$\boxed{\Gamma, s \vdash\ T\ \mathbf{wf}}$ Well-formed types

$$\frac{\begin{array}{l}\Gamma, s \vdash\ T\ \mathbf{wfe} \\ \Gamma, z : T\{z \Rightarrow \overline{D}\}, s \vdash\ \overline{D}\ \mathbf{wf}\end{array}}{\Gamma, s \vdash\ T\{z \Rightarrow \overline{D}\}\ \mathbf{wf}}\ \text{WF\_RFN}$$

$$\frac{\Gamma, s \vdash p \ni L : S..U \quad \Gamma, s \vdash S\,\mathbf{wfe} \quad \Gamma, s \vdash U\,\mathbf{wfe}}{\Gamma, s \vdash p.L\,\mathbf{wf}}\ \text{WF\_TSEL1}$$

$$\frac{\Gamma, s \vdash p \ni L : \bot..U}{\Gamma, s \vdash p.L\,\mathbf{wf}}\ \text{WF\_TSEL2}$$

$$\frac{\Gamma, s \vdash T\,\mathbf{wfe} \quad \Gamma, s \vdash T'\,\mathbf{wfe}}{\Gamma, s \vdash T \wedge T'\,\mathbf{wf}}\ \text{WF\_AND}$$

$$\frac{\Gamma, s \vdash T\,\mathbf{wfe} \quad \Gamma, s \vdash T'\,\mathbf{wfe}}{\Gamma, s \vdash T \vee T'\,\mathbf{wf}}\ \text{WF\_OR}$$

$$\frac{}{\Gamma, s \vdash \bot\,\mathbf{wf}}\ \text{WF\_BOT}$$

$$\frac{}{\Gamma, s \vdash \top\,\mathbf{wf}}\ \text{WF\_TOP}$$

$\boxed{\Gamma, s \vdash D\,\mathbf{wf}}$ Well-formed declarations

$$\frac{\Gamma, s \vdash S\,\mathbf{wfe} \quad \Gamma, s \vdash U\,\mathbf{wfe}}{\Gamma, s \vdash L : S..U\,\mathbf{wf}}\ \text{DECL\_WF\_TYPE}$$

$$\frac{\Gamma, s \vdash T\,\mathbf{wfe}}{\Gamma, s \vdash l : T\,\mathbf{wf}}\ \text{DECL\_WF\_VALUE}$$

$$\frac{\Gamma, s \vdash S\,\mathbf{wfe} \quad \Gamma, s \vdash T\,\mathbf{wfe}}{\Gamma, s \vdash m : S \to T\,\mathbf{wf}}\ \text{DECL\_WF\_METHOD}$$

$\boxed{\Gamma, s \vdash \overline{D}\,\mathbf{wf}}$

$\boxed{\Gamma, s \vdash T\,\mathbf{wfe}}$ Well-formed and expanding types

$$\frac{\Gamma, s \vdash T\,\mathbf{wf} \quad \Gamma, s \vdash T \prec_z \overline{D}}{\Gamma, s \vdash T\,\mathbf{wfe}}\ \text{WFE\_ANY}$$

$\boxed{\Gamma \models s}$ Consistent Context

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Definition rules:        53 good    0 bad
Definition rule clauses: 141 good    0 bad
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