

# Foundations of Software

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Master level course, 2007

September 18, 2007 - version 1.1

Slides in part adapted from:  
University of Pennsylvania CIS 500: Software Foundations - Fall 2006  
by Benjamin Pierce

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## Course Overview

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## What is “software foundations”?

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Software foundations (or “theory of programming languages”) is the mathematical study of the **meaning** of programs.

The goal is finding ways to describe program behaviors that are both **precise** and **abstract**.

- ▶ **precise** so that we can use mathematical tools to formalize and check interesting properties
- ▶ **abstract** so that properties of interest can be discussed clearly, without getting bogged down in low-level details

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## Why study software foundations?

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- ▶ To prove specific properties of particular programs (i.e., program verification)
  - ▶ Important in some domains (safety-critical systems, hardware design, security protocols, inner loops of key algorithms, ...), but still quite difficult and expensive
- ▶ To develop intuitions for *informal* reasoning about programs
- ▶ To prove general facts about all the programs in a given programming language (e.g., safety or isolation properties)
- ▶ To understand language features (and their interactions) deeply and develop principles for better language design  
(PL is the “materials science” of computer science...)

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## What you can expect to get out of the course

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- ▶ A more sophisticated perspective on programs, programming languages, and the activity of programming
  - ▷ How to view programs and whole languages as formal, mathematical objects
  - ▷ How to make and prove rigorous claims about them
  - ▷ Detailed study of a range of basic language features
- ▶ Deep intuitions about key language properties such as type safety
- ▶ Powerful tools for language design, description, and analysis

Most software designers are language designers!

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## What this course is not

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- ▶ An introduction to programming
- ▶ A course on functional programming (though we'll be doing some functional programming along the way)
- ▶ A course on compilers (you should already have basic concepts such as lexical analysis, parsing, abstract syntax, and scope under your belt)
- ▶ A comparative survey of many different programming languages and styles

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## Approaches to Program Meaning

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- ▶ **Denotational semantics** and **domain theory** view programs as simple mathematical objects, abstracting away their flow of control and concentrating on their input-output behavior.
- ▶ **Program logics** such as **Hoare logic** and **dependent type theories** focus on logical rules for reasoning about programs.
- ▶ **Operational semantics** describes program behaviors by means of abstract machines. This approach is somewhat lower-level than the others, but is extremely flexible.
- ▶ **Process calculi** focus on the communication and synchronization behaviors of complex concurrent systems.
- ▶ **Type systems** describe approximations of program behaviors, concentrating on the shapes of the values passed between different parts of the program.

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## Overview

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This course will concentrate on operational techniques and type systems.

- ▶ Part I: Modeling programming languages
  - ▷ Syntax and parsing
  - ▷ Operational semantics
  - ▷ Inductive proof techniques
  - ▷ The lambda-calculus
  - ▷ Syntactic sugar; fully abstract translations
- ▶ Part II: Type systems
  - ▷ Simple types
  - ▷ Type safety
  - ▷ References
  - ▷ Subtyping

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## Overview

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- ▶ Part III: Object-oriented features (case study)
  - ▷ A simple imperative object model
  - ▷ An analysis of core Java
  - ▷ An analysis of core Scala

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## Organization of the Course

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## People

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Instructor: Martin Odersky  
INR 319  
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Co-instructor: Erik Ernst  
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Teaching Assistants: Iulian Dragos  
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<gilles.dubochet@epfl.ch>

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## Information

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Textbook: Types and Programming Languages,  
Benjamin C. Pierce, MIT Press, 2002

Webpage: [http://lampwww.epfl.ch/teaching/foundations\\_of\\_software/](http://lampwww.epfl.ch/teaching/foundations_of_software/)

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## Elements of the Course

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- ▶ The Foundations of Software course consists of
  - ▷ lecture (Tuesday 10:15-12:00, room INM 201)
  - ▷ exercises and project work (Wednesday 10:15-12:00, rooms CO 020, CO 121)
- ▶ The lecture will follow in large parts the textbook.
- ▶ For lack of time, we cannot treat all essential parts of the book in the lectures, that's why the [textbook is required reading](#) for participants of the course.

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## Homework and Projects

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You will be asked to

- ▶ solve and hand in some written exercise sheets,
- ▶ do a number of programming assignments, including
  - ▷ parsers,
  - ▷ interpreters and reduction engines,
  - ▷ type checkersfor a variety of small languages.
- ▶ The recommended implementation language for these assignments is [Scala](#).

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## Scala

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- ▶ Scala is a functional and object-oriented language that is closely interoperable with Java.
- ▶ It is very well suited as an implementation language for type-checkers, in particular because it supports:
  - ▷ pattern matching,
  - ▷ higher-order functions,
  - ▷ inheritance and mixins.

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## Learning Scala

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If you don't know Scala yet, there's help:

- ▶ The Scala web site:  
[www.scala-lang.org](http://www.scala-lang.org)
- ▶ On this site, the documents:
  - ▷ *A Brief Scala Tutorial - an introduction to Scala for Java programmers.* (short and basic).
  - ▷ *An Introduction to Scala* (longer and more comprehensive).
  - ▷ *An Overview of the Scala Programming Language* (high-level).
  - ▷ *Scala By Example* (long, comprehensive, tutorial style).
- ▶ The assistants.

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## Grading and Exams

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Final course grades will be computed as follows:

- ▶ Homework and project: 30%
- ▶ Mid-term exam: 30%
- ▶ Final exam: 40%

### Exams:

1. Mid-term: Tue, Nov 13th, 2007
2. Final exam: Fri, Dec 21st, 2007

(dates are provisional)

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## Collaboration

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- ▶ Collaboration on homework is **strongly encouraged**.
- ▶ Studying with other people is the best way to internalize the material
- ▶ Form pair programming and study groups!  
2-3 people is a good size. 4 is too many for all to have equal input.

"You never really misunderstand something  
until you try to teach it...  
" – Anon.

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## Plagiarism

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- ▶ A single group will of course share code.
- ▶ But plagiarizing **code** by **other groups** as part of a project is unethical and will not be tolerated, whatever the source.

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## Part I

## Modelling programming languages

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## Syntax and Parsing

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- ▶ The first-level of modeling a programming language concerns its [context-free syntax](#).
- ▶ Context free syntax determines a set of legal [phrases](#) and determines the [\(tree-\)structure](#) of each of them.
- ▶ It is often given on two levels:
  - ▷ [concrete](#): determines the exact (character-by-character) set of legal phrases
  - ▷ [abstract](#): concentrates on the tree-structure of legal phrases.
- ▶ We will be mostly concerned with abstract syntax in this course.
- ▶ But to be able to write complete programming tools, we need a convenient way to map character sequences to trees.

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## Approaches to Parsing

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There are two ways to construct a parser:

- ▶ [By hand](#) Derive a parser program from a grammar.
- ▶ [Automatic](#) Submit a grammar to a tool which generates the parser program.

In the second approach, one uses a special [grammar description language](#) to describe the input grammar.

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## Domain-Specific Languages

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- ▶ The grammar description language is an example of a [domain-specific language \(DSL\)](#).
- ▶ The parser generator acts as a processor ([“compiler”](#)) for this language — that’s why it’s sometimes called grandly a [“compiler-compiler”](#).
- ▶ Example of a “program” in the grammar description DSL:

```
Expr ::= Term { '+' Term | '-' Term }.  
Term ::= Factor { '*' Factor | '/' Factor }.  
Factor ::= Number | '(' Expr ')'
```

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## Hosted Domain Specific Languages

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- ▶ An alternative to a stand-alone DSL is a [hosted DSL](#).
- ▶ Here, the DSL does not exist as a separate language but as an API in a [host language](#).
- ▶ The host language is usually a general purpose programming language.

We will now develop this approach for grammar description languages.

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## A Hosted Grammar Description Language in Scala

We will develop a framework where grammars can be described like this:

```
def expr : Parser[Any] = term ~ rep("+ ~ term | - ~ term)
def term : Parser[Any] = factor ~ rep("* ~ factor | "/" ~ factor)
def factor : Parser[Any] = "(" ~ expr ~ ")" | numericLit
```

This description can be produced from the previous grammar by systematic text replacements:

- ▶ Insert a `def` at the beginning of each production.
- ▶ The “`::=`” becomes “`: Parser[Any] =`”.
- ▶ Sequential composition is now expressed by a `~`.
- ▶ Repetition `{...}` is now expressed by `rep(...)`.
- ▶ Option `[...]` is now expressed by `opt(...)`.
- ▶ The point at the end of a production is removed.

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## Parser Combinators

- ▶ The differences between Grammar A and Grammar B are fairly minor.  
(Note in particular that existing DSL's for grammar descriptions also tend to add syntactic complications to the idealized Grammar A we have seen).
- ▶ The important difference is that Grammar B is a valid Scala program, when combined with an API that defines the necessary primitives.
- ▶ These primitives are called `parser combinators`.

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## The Basic Idea

For each language (identified by grammar symbol  $S$ ), define a function  $f_S$  that, given an input stream  $i$ ,

- ▶ if a prefix of  $i$  is in  $S$ , return `Success(Pair( $x$ ,  $i'$ ))` where  $x$  is a result for  $S$  and  $i'$  is the rest of the input.
- ▶ otherwise, return `Failure(msg,  $i$ )` where `msg` is an error message string.

The first behavior is called `success`, the second `failure`.

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## The Basic Idea in Code

```
Assume:
class StandardTokenParsers {
  type Parser = Input => ParseResult
}
```

where

```
type Input = Reader[Token] // a stream of Tokens with positions.
```

and we assume a class `Token` with subclasses

- ▶ `case class Keyword(chars: String)` for keywords,
- ▶ `case class NumericLit(chars: String)` for numbers,
- ▶ `case class StringLit(chars: String)` for strings,
- ▶ `case class Identifier(chars: String)` for identifiers.

In each case, `chars` represents the characters making up the token.

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Also assume a class `ParseResult[T]` with subclasses

```
case class Success[T](result: T, in: Input)
  extends ParseResult[T]
case class Failure(msg: String, in: Input)
  extends ParseResult[Nothing]
```

## Object-Oriented Parser Combinators

- ▶ In fact, we will also need to express `|` and `~` as methods of parsers.
- ▶ That's why we extend the function type of parsers as follows:

```
abstract class Parser extends (Input => ParseResult[T]) {
  // An unspecified method that defines the parser function.
  def apply(in: Input): ParseResult

  // A parser combinator for sequential composition
  def ~ ...

  // A parser combinator for alternative composition
  def | ...
}
```

It remains to define concrete combinators that implement this class (see below).

## A Generic Single-Token Parser

- ▶ The following parser succeeds if the first token in the input satisfies a given predicate `p`.<sup>a</sup>

- ▶ If it succeeds, it reads the token string and returns it as a result.  

```
def token(kind: String)(p: Tokens => Boolean) = new Parser[String] {
  def apply(in: Input) =
    if (p(in.head)) Success(in.head.chars, in.tail)
    else Failure(kind+" expected", in)
}
```

<sup>a</sup>This is somewhat of a simplification; the actual framework contains an `elem` parser instead which returns the token itself

## Specific Single-Token Parsers

- ▶ The following parser succeeds if the first token in the input is a given keyword "chars":

- ▶ If it succeeds, it returns a keyword token as a result.  

```
implicit def keyword(chars: String) = token("+"chars+"") {
  case Keyword(chars1) => chars == chars1
  case _ => false
}
```

- ▶ Note that `keyword` is defined `implicit`. This means we can usually just write (e.g.) `"if"` for `keyword("if")`.



- ▶ The following parsers succeed if, respectively, the first token in the input is a numeric or string literal or an identifier.
 

```
def numericLit = token("number")(_.isInstanceOf[NumericLit])
def stringLit = token("string literal")(_.isInstanceOf[StringLit])
def ident = token("identifier")(_.isInstanceOf[Identifier])
```

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## The Sequence Combinator

- ▶ The sequence combinator  $P \sim Q$  succeeds if  $P$  and  $Q$  both succeed. It then returns a list containing the concatenation of result of  $P$  and the result of  $Q$ .

- ▶  $\sim$  is implemented as a method of class `Parser`.

```
case class Parser[T] {
  def ~ [U](q: Parser[U]) = new Parser[T ~ U] {
    def apply(in: Input) = Parser.this(in) match {
      case Success(x, in1) =>
        q(in1) match {
          case Success(y, in2) => Success(new ~(x, y), in2)
          case failure => failure
        }
      case failure => failure
    }
  }
}
```

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## Concatenating Results

Normally, the  $\sim$  combinator returns a the results of the two parsers that are run sequentially wrapped in a  $\sim$ -object.

Here  $\sim$  is also case class that concatenates two results

```
case class ~(T, U)(_1: T, _2: U) {
  override def toString = "(" + _1 + " ~ " + _2 + ")"
}
```

(one could have also used a pair for this, but  $\sim$  turns out to be nicer – see below).

There are also two variants of  $\sim$  which return only the left or only the right operand.

```
P <~ Q // returns only result of P
P ~> Q // returns only result of Q
```

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## The Alternative Combinator

- ▶ The alternative combinator  $P | Q$  succeeds if either  $P$  or  $Q$  succeeds.
- ▶ It returns the result of  $P$  if  $P$  succeeds, or the result of  $Q$ , if  $Q$  succeeds.
- ▶ The alternative combinator is implemented as a method of class `Parser`.

```
def | (q: => Parser[T]) = new Parser[T] {
  def apply(in: Input) = Parser.this(in) match {
    case s1 @ Success(_, _) => s1
    case failure => q(in)
  }
}
```

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## Failure And Success Parsers

- ▶ The parser `failure(msg)` always fails with the given error message. It is implemented as follows:

```
def failure(msg: String) = new Parser[Nothing] {
  def apply(in: Input) = Failure(msg, in)
}
```

- ▶ The parser `success(result)` always succeeds with the given result. It does not consume any input. It is implemented as follows:

```
def success(result: T) = new Parser[T] {
  def apply(in: Input) = Success(result, in)
}
```

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## Result Conversion

The parser  $P \Rightarrow f$  succeeds iff  $P$  succeeds. In that case it returns the result of applying  $f$  to the result of  $P$ .

```
def [U](f: T => U) = new Parser[U] {
  def apply(in: Input) = Parser.this(in) match {
    case Success(x, in1) => Success(f(x), in1)
    case f => f
  }
}
```

A variant  $\Rightarrow\Rightarrow$  takes a value  $V$  as right hand side argument.

It returns  $V$  if the left hand parser succeeds:

```
def [U](r: U): Parser[U] = (x => r)
```

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## Option and Repetition Combinators

- ▶ The `opt( $P$ )` combinator always succeeds and returns an `Option` result. It returns `Some( $R$ )` iff  $P$  succeeds with  $R$ , and it returns `None` if  $P$  fails.
- ▶ The `rep( $P$ )` combinator applies  $P$  zero or more times until  $P$  fails. It returns a list of all results returned by  $P$ .

The two combinators are implemented as follows:

```
def opt[T](p: Parser[T]): Parser[Option[T]] =
  p ^^ Some | success(None)
def rep[T](p: Parser[T]): Parser[List[T]] =
  p ~ rep(p) ^^ { case x ~ xs => x :: xs } | success(List())
```

Note that neither of these combinators can fail!

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## The Interleaved Repetition Combinator

The `repsep( $P, Q, \dots, Q, P$ )` parser parses a (possibly empty) sequence

It returns a list of all results returned by  $P$ .

**Example:** `repsep(ident, ",")` parses a list of identifiers separated by commas.

The `repsep` combinator is implemented as follows:

```
def repsep[T, U](p: Parser[T], q: Parser[U]): Parser[List[T]] =
  p ~ rep(q ~> p) ^^ { case r ~ rs => r :: rs } | success(List())
```

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## Other Combinators

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More combinators can be defined if necessary.

**Exercise:** Implement the `rep1(P)` parser combinator, which applies  $P$  one or more times.

**Exercise:** Define `opt` and `rep` directly, without making use of `~`, `|`, and `empty`.

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## An Example: JSON

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JSON, or JavaScript Object Notation, is a popular data interchange format.

JSON data essentially consists of objects `{...}`, arrays `[...]`, numbers, and strings.

Here is an example of a JSON value:

```
{ "address book": {  
  "name": "John Smith",  
  "address": { "street": "10 Market Street",  
              "city"  : "San Francisco, CA",  
              "zip"   : 94111 },  
  "phone numbers": ["408 338-4238", "408 111-6892"]  
}}
```

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## A JSON parser

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```
package examples.parsing  
  
// import the standard parser class  
import scala.util.parsing.combinator1.syntactical.StandardTokenParsers  
  
object JSON extends StandardTokenParsers {  
  // fix some delimiter symbols ...  
  lexical.delimiters += ("{" , "}", "[" , "]", ":" )  
  // ... and some reserved words  
  lexical.reserved += ("null", "true", "false")  
  
  // here are the four productions making up the JSON grammar  
  def obj  : Parser[Any] = "{" ~ repsep(member, ",") ~ "}"  
  def arr  : Parser[Any] = "[" ~ repsep(value, ",") ~ "]"  
  def member : Parser[Any] = stringLit ~ ":" ~ value  
  def value : Parser[Any] = stringLit | numericLit | obj | arr |  
    "null" | "true" | "false"
```

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## Testing the JSON Parser

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Add a method `main` that can be used to test the parser.

```
def main(args: Array[String]) {  
  val tokens = new lexical.Scanner(args(0))  
  println(args(0))  
  println(phrase(value)(tokens))  
}
```

Here are two test runs:

```
> java examples.parsing.JSON "{ \"x\": true, \"y\": [1, 2, 3] }"  
{ x: true, y: [1, 2, 3] }  
[1.26] parsed: ((({ ~ List(((x ~ : ) ~ true), ((y ~ :) ~ (([ ~ List(1,  
2, 3)) ~ ))))) ~ ))
```

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```
>java examples.parsing.JSON "{ \"x\": true \"y\": [1, 2] }"
{ x: true y: [1, 2] }
[1.13] failure: unexpected token string literal y
{ \"x\": true \"y\": [1, 2] }
```

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## Getting Better Output

- ▶ The result of the previous JSON parser was a tree containing all input tokens (in some not very legible form).
- ▶ We can get a more useful result by adding ^^ parts to the productions:

```
def obj : Parser[Any] = // return a Map
  "{" ~> rep(member) <~ "}" ^^ (ms => Map() ++ ms)

def arr : Parser[Any] = // return a List
  "[" ~> rep{value} <~ "]"

def member : Parser[Any] = // return a name/value pair
  stringLit ~ ":" ~ value ^^
  { case name ~ ":" ~ value => (name, value) }
```

...

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```
def value : Parser[Any] = (
  | obj
  | arr
  | stringLit
  | numericLit ^^ (...toInt) // return an Int
  | "null" ^^ null // return 'null'
  | "true" ^^ true // return 'true'
  | "false" ^^ false // return 'false'
)
```

If we run the test now, we get:

```
>java examples.parsing.JSON1 "{ \"x\": true, \"y\": [1, 2, 3] }"
{ x: true, y: [1, 2, 3] }
[1.30] parsed: Map(x → true, y → List(1, 2, 3))
```

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## Table of Parser Combinators

ident	identifier
keyword(...)	keyword or special symbol (implicit)
numericLit	integer number
stringLit	string literal
P ~ Q	sequential composition
P <~ Q, P ~> Q	sequential composition; keep left/right only
P   Q	alternative
opt(P)	option
rep(P)	repetition
repsep(P, Q)	interleaved repetition
P ^^ f	result conversion
P ^^ v	constant result

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## Arithmetic Expressions Again

Here is the full parser for arithmetic expressions:

```
object Arithmetic extends StandardTokenParsers {
  lexical.delimiters += List("(", ")", "+", "-", "*", "/")
  def expr: Parser[Any] = term ~ rep("+ ~ term | - ~ term)
  def term = factor ~ rep("* ~ factor | / ~ factor)
  def factor: Parser[Any] = "(" ~ expr ~ ")" | numericLit
```

**Question:** How can we make it evaluate the parsed expression?

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## A Problem with Top-Down Parsing

Because Parser Combinators work top-down, they do not allow left-recursion.

A production like  
`def expr = expr ~ "-" ~ term`

would go into an infinite recursion when executed.

The alternative:  
`def expr = term ~ rep("- ~ term)`

produces a "right-leaning" tree:  $X - Y - Z$  parses  
 $X \text{ List}(- Y, - Z)$

But the correct reduction/evaluation of  $+, -, *, /$  is left-leaning!

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## Evaluate by FoldLeft

We can solve this problem by delaying reduction until all elements of a repetition have been parsed and then performing a fold left on the list:

```
def expr: Parser[Int] =
  term ~ rep("+ ~ term | - ~ term) ^^ reduceList
def term: Parser[Int] =
  factor ~ rep("* ~ factor | / ~ factor) ^^ reduceList
def factor: Parser[Int] =
  "(" ~> expr <~ ")" | numericLit ^^ (_.toInt)
```

Here, `reduceList` is defined in terms of the fold-left operation `/:` ...

```
val reduceList: Expr ~ List[String ~ Expr] => Expr = {
  case i ~ ps => (i /: ps)(reduce)
}
```

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... and `reduce` is defined as follows:

```
def reduce(x: Int, r: String ~ Int) = r match {
  case "+" ~ y => x + y
  case "-" ~ y => x - y
  case "*" ~ y => x * y
  case "/" ~ y => x / y
  case _ => throw new MatchError("illegal case: " + r)
}
```

With this, we get:

```
java examples.parsing.ArithmeticParsers1 "2 * (3 + 7)"
2 * (3 + 7)
[1.12] parsed: 20
```

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## Conclusion

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- ▶ Combinator parsers give a provide a concise, flexible, and high-level way to construct parsers.
- ▶ The token classes of a context free grammar are modelled as primitive parsers.
- ▶ The combination forms are modelled as higher-order parsers.
- ▶ Combinator parsers are an example of an embedded DSL.
- ▶ By contrast, classical parser generators can be classified as stand-alone DSLs.
- ▶ Advantage of an embedded DSL over a parser generator: It's easier to connect the results of combinator parsers with the environment.
- ▶ Disadvantage: lower efficiency – but this can be overcome.