

## Independently Extensible Solutions to the Expression Problem

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## History

The **expression problem** is fundamental for software extensibility.

- It arises when recursively defined datatypes and operations on these types have to be extended simultaneously.
- It was first stated by Cook [91], named as such by Wadler [98].
- Many people have worked on the problem since.

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## Problem Statement

Suppose we have

- a recursively defined **datatype**, defined by a set of cases, and
- **processors** which operate on this datatype.

There are two directions which we can extend this system:

1. Extend the datatype with new **data variants**,
2. Add new **processors**.

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## Problem Statement (2)

Find an implementation technique which satisfies the following:

- **Extensibility in both dimensions**: It should be possible to add new data variants *and* processors.
- **Strong static type safety**: It should be impossible to apply a processor to a data variant which it cannot handle.
- **No modification or duplication**: Existing code should neither be modified nor duplicated.
- **Separate compilation**: Compiling datatype extensions or adding new processors should not encompass re-type-checking the original datatype or existing processors.

New concern in this paper:

- **Independent extensibility**: It should be possible to combine independently developed extensions so that they can be used jointly.

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## Scenario

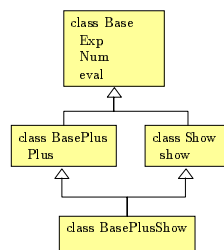
Say, we have

- a base type `Exp` for expressions, with an operation `eval`.
- a concrete subtype `Num` of `Exp`, representing integer numbers.

We now want to extend this system with

- a new expression type: `Plus`
- a new operation: `show`

Finally, we want to combine both extensions in one system.



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## State of the Art 10 Years Ago

Two canonical structuring schemes each support extension in one dimension, but prevent extension in the other.

1. A data centric structure (using virtual methods) enables addition of new kinds of data.
2. An operation centric structure (using pattern matching or visitors) enables addition of new kinds of operations.

More refined solutions often build on one of these schemes.

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## State of the Art Today

Many people have proposed partial solutions to the expression problem:

- By allowing a certain amount of dynamic typing or reflection: extensible visitors (Krishnamurti, Felleisen, Friedman 98), walkabouts (Palsberg and Jay 97).
- By allowing default behavior: multi-methods (MultiJava 2000), visitors with defaults (Odersky, Zenger 2001).
- By deferring type checking to link time (relaxed MultiJava 2003).
- Using *polymorphic variants* (Garrigue 2000)
- Using *ThisType* and *matching* (Bruce 2003)
- Using *generics* with some clever tricks (Torgersen 2004)

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## In this Paper

- We present new solutions to the expression problem.
- They satisfy all the criteria mentioned above, including independent extensibility.
- We study two new variations of the problem: tree transformers and binary methods.
- Two families of solutions: data-centric and operation-centric. Each one is the dual of the other.

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- Our solutions are written in Scala.
- They make essential use of the following language constructs:
  - abstract types,
  - mixin composition, and
  - explicit self types (for the visitor solution).
 (These are also the core constructs of the *vObj* calculus).
- Compared to previous solutions, ours tend to be quite concise.
- These solutions were also reproduced in OCaml (Rémy 2004).

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## To Default or Not Default?

- Solutions to the expression problem fall into two broad categories - with defaults and without.
- Solutions with defaults permit processors that handle unknown data uniformly, using a default case.
- Such solutions tend to require less planning.
- However, often no useful behavior for a default case exists, there's nothing a processor known to do except throw an exception.
- This is re-introduces run-time errors through the backdoor.

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## A Solution with Defaults

Outer trait defines system in question  
Everything else is nested in it.

Base language:

```
trait Base {
  class Exp;
  case class Num(v: Int)
  extends Exp
  def eval(e: Exp) = e match {
    case Num(v) => v
  }
}
```

Operation extension:

```
trait Show extends Base {
  def show(e: Exp) = e match {
    case Num(v) => v.toString()
  }
}
```

```
trait BasePlus extends Base {
  case class Plus(l: Exp, r: Exp)
  extends Exp;
  def eval(e: Exp) = e match {
    case Plus(l, r) => eval(l) + eval(r)
    case _ => super.eval(e)
  }
}
```

Comments:

what if we had forgotten  
to override show?

```
trait ShowPlus extends Show with BasePlus {
  override def show(e: Exp) = e match {
    case Plus(l, r) => show(l) + "+" + show(r)
    case _ => super.show(e)
  }
}
```

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## Solutions without Defaults

Solutions without defaults fall into two categories.

- **Data-centric:** operations are distributed as methods in the individual data types.
- **Operation-centric:** operations are grouped separately in a visitor object.

Let's try data-centric first.

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**Problem:** type Exp needs to vary co-variantly with operation extensions.

**Solution**

```

Base language:
trait Base {
  trait Exp { def eval: int }
  class Num(v: int) extends Exp {
    val value = v;
    def eval = value
  }
}

Operation extension:
trait Show extends Base {
  trait Exp extends super.Exp {
    def show: String;
  }
  class Num(v: int) extends
    super.Num(v) with Exp {
    def show = value.toString();
  }
}

Combined extension:
trait BasePlus extends Base {
  class Plus(l: Exp, r: Exp)
  extends Exp {
    val left: Exp = l;
    val right: Exp = r;
    def eval = left.eval + right.eval
  }
}

trait ShowPlus extends BasePlus with Show {
  class Plus(l: Exp, r: Exp) extends
    super.Plus(l, r) with Exp {
    def show =
      left.show + "+" + right.show;
  }
}

```

**ERROR:** show is not a member of Base.Exp!

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### Achieving Covariance

- Covariant adaptation can be achieved by defining an *abstract type*.
- Example:
 

```
type exp <: Exp;
```

 This defines exp to be an abstract type with upper bound Exp.
- The exp type can be refined co-variantly in subtypes.

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### Data-centric Solution

Base language:

```

trait Base {
  type exp <: Exp;
  trait Exp { def eval: int }
  class Num(v: int) extends Exp {
    val value = v;
    def eval = value
  }
}

```

Operation extension:

```

trait Show extends Base {
  type exp <: Exp;
  trait Exp extends super.Exp {
    def show: String;
  }
  class Num(v: int) extends
    super.Num(v) with Exp {
    def show = value.toString();
  }
}

```

Data extension:

```

trait BasePlus extends Base {
  class Plus(l: exp, r: exp)
  extends Exp {
    val left: exp = l;
    val right: exp = r;
    def eval = left.eval + right.eval
  }
}

```

Combined extension:

```

trait ShowPlus extends BasePlus with Show {
  class Plus(l: exp, r: exp) extends
    super.Plus(l, r) with Exp {
    def show =
      left.show + "+" + right.show;
  }
}

```

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### Tying the Knot

- Classes that contain abstract types are themselves abstract.
- Before instantiating such a class, the abstract type has to be defined concretely.
- This is done using a type alias, e.g. type exp = Exp;
- For instance, here is a test program that uses the ShowPlus system.

```

object ShowPlusTest extends ShowPlusNeg with Application {
  type exp = Exp;
  val e: Exp = new Plus(new Num(1), new Num(2));
  Console.println(e.show + " = " + e.eval)
}

```

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### Independent Data Extensions

- Let's add to the system with eval and show another data variant for negated terms.

```

trait ShowNeg extends Show {
  class Neg(t: exp) extends Exp {
    val term = t;
    def eval = - term.eval;
    def show = "-" + term.show + " "
  }
}

```

- The two extensions ShowPlus and ShowNeg can be combined using a simple mixin composition:

```

trait ShowPlusNeg extends ShowPlus with ShowNeg;

```

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### Tree Transformer Extensions

- So far, all our operators returned simple data types.
- We now study tree transformers, i.e. operators that return themselves the data structure in question.
- This is in principle as before, except that we need to add factory methods.
- As an example, consider adding an operation dble that, given an expression tree of value v, returns another tree that evaluates to 2\*v.

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## The "Dble" Transformer

```

trait DblePlus extends BasePlus {
  type exp <: Exp;
  trait Exp extends super.Exp {
    def dble: exp;
  }
  def Num(v: Int): exp;
  def Plus(l: exp, r: exp): exp;

  class Num(v: Int) extends super.Num(v) with Exp {
    def dble = Num(v * 2);
  }
  class Plus(l: exp, r: exp) extends super.Plus(l, r) with Exp {
    def dble = Plus(left.dble, right.dble);
  }
}

```

Factory methods

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## Combining "Show" and "Dble"

- Combining two operations is more complicated than a simple mixin composition.
- We now have to combine as well all nested types in a "deep composition".

```

trait ShowDblePlus extends ShowPlus with DblePlus {
  type exp <: Exp;

  trait Exp extends super[ShowPlus].Exp
    with super[DblePlus].Exp;

  class Num(v: Int) extends super[ShowPlus].Num(v)
    with super[DblePlus].Num(v)
    with Exp;

  class Plus(l: exp, r: exp) extends super[ShowPlus].Plus(l, r)
    with super[DblePlus].Plus(l, r)
    with Exp;
}

```

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## Instantiating Transformers

- Instantiating a system with transformers works as before, except that we now also need to define factory methods.

```

trait ShowDblePlusTest extends ShowDblePlus with Application {
  type exp = Exp;

  def Num(v: Int) = new Num(v);
  def Plus(l: exp, r: exp): exp = new Plus(l, r);

  val e: exp = new Plus(new Num(1), new Num(2));
  Console.println(e.dble.eval);
}

```

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## Summary: Data-centric solutions

- We have seen that we can flexibly extend in two dimensions using a data-centric approach.
- Extension with new operations is made possible by abstracting over the data type `exp`.
- Individual extensions can be merged later using mixin composition.
- Merging two data extensions is easy, requires only a flat mixin composition.
- Merging two operation extensions is harder, since it requires to merge nested classes as well, using a deep mixin composition.

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## Operation-centric Solutions

- Operation-centric solutions are the duals of data-centric solutions.
- Here, all operations together are grouped in a *visitor object*.

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## Operation-centric Solution

Base language:

```

trait Base {
  trait Exp { def accept(v: Visitor): Unit }

  class Num(value: Int) extends Exp {
    def accept(v: Visitor): Unit = v.visitNum(value);
  }

  type Visitor <: Visitor;
  trait Visitor {
    def visitNum(value: Int): Unit;
  }

  class Eval[Visitor] extends Visitor {
    var result: Int = 0;
    def apply(t: Exp): Int = { t.accept(this); result }
    def visitNum(value: Int): Unit = { result = value }
  }
}

```

Solution:  
explicit self type

Problem:  
Eval.this must conform to Visitor

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## Selftype Annotations

- Scala is one of very few languages where the type of this can be fixed by the programmer using a selftype annotation (OCaml is another).
- Type-soundness is maintained by two requirements
  - Selftypes vary covariantly in the class hierarchy.
    - I.e. the selftype of a class must be a subtype of the selftypes of all its superclasses.
    - Classes that are instantiated to objects must conform to their selftypes.
- Selftype annotations are not the same thing as Bruce's *mytype*, since they do not vary automatically.

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## Operation-centric Solution (2)

Base language:

```
trait Base {
  trait Eval { def accept(v: Visitor): Unit }
  class Num(value: Int) extends Exp {
    def accept(v: Visitor): Unit = v.visitNum(value)
  }
  type Visitor <- Visitor
  trait Visitor {
    def visitNum(value: Int): Unit
  }
  class Plus(left: Exp, right: Exp) extends Exp {
    var result: Int = _
    def apply(): Exp { Int } = { t.accept(this); result }
    def visitNum(value: Int): Unit = { result = value }
  }
}
```

Data extension:

```
trait BasePlus extends Base {
  type visitor <- Visitor;
  trait Visitor extends super.Visitor {
    def visitPlus(left: Exp, right: Exp): Unit;
  }
  class Plus(left: Exp, right: Exp) extends Exp {
    def accept(v: visitor): Unit =
      v.visitPlus(left, right);
  }
  class Eval: visitor extends
    super.Eval with Visitor {
    def visitPlus(l: Exp, r: Exp): Unit =
      result = apply(l) + apply(r);
  }
}
```

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## Operation-centric Solution (3)

Base language:

```
trait Base {
  trait Eval { def accept(v: Visitor): Unit }
  class Num(value: Int) extends Exp {
    def accept(v: Visitor): Unit = v.visitNum(value)
  }
  type Visitor <- Visitor;
  trait Visitor {
    def visitNum(value: Int): Unit
  }
  class Eval: visitor extends Visitor {
    var result: Int = _
    def apply(): Exp { Int } = { t.accept(this); result }
    def visitNum(value: Int): Unit = { result = value }
  }
}
```

Data extension:

```
trait BasePlus extends Base {
  type visitor <- Visitor;
  trait Visitor extends super.Visitor {
    def visitPlus(left: Exp, right: Exp): Unit;
  }
  class Plus(left: Exp, right: Exp) extends Exp {
    def accept(v: visitor): Unit =
      v.visitPlus(left, right);
  }
  class Eval: visitor extends
    super.Eval with Visitor {
    def visitPlus(l: Exp, r: Exp): Unit =
      result = apply(l) + apply(r);
  }
}
```

Operation extension:

```
trait Show extends Base {
  class Show: visitor extends Visitor {
    var result: String = _;
    def apply(t: Exp): String = { t.accept(this); result }
    def visitNum(value: Int): Unit =
      { result = value.toString() }
  }
}
```

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## Operation-centric Solution (4)

Base language:

```
trait Base {
  trait Eval { def accept(v: Visitor): Unit }
  class Num(value: Int) extends Exp {
    def accept(v: Visitor): Unit = v.visitNum(value)
  }
  type Visitor <- Visitor;
  trait Visitor {
    def visitNum(value: Int): Unit
  }
  class Eval: visitor extends Visitor {
    var result: Int = _
    def apply(): Exp { Int } = { t.accept(this); result }
    def visitNum(value: Int): Unit = { result = value }
  }
}
```

Data extension:

```
trait BasePlus extends Base {
  type visitor <- Visitor;
  trait Visitor extends super.Visitor {
    def visitPlus(left: Exp, right: Exp): Unit;
  }
  class Plus(left: Exp, right: Exp) extends Exp {
    def accept(v: visitor): Unit =
      v.visitPlus(left, right);
  }
  class Eval: visitor extends
    super.Eval with Visitor {
    def visitPlus(l: Exp, r: Exp): Unit =
      result = apply(l) + apply(r);
  }
}
```

Operation extension:

```
trait Show extends Base {
  class Show: visitor extends Visitor {
    var result: String = _;
    def apply(): Exp { String } = { t.accept(this); result }
    def visitNum(value: Int): Unit =
      { result = value.toString() }
  }
}
```

Combined extension:

```
trait ShowPlus extends Show with BasePlus {
  class Show: visitor extends super.Show {
    def visitPlus(l: Exp, r: Exp): Unit =
      result = apply(l) + "+" + apply(r);
  }
}
```

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## Summary: Operation-centric solutions

- Operation-centric is the dual of data-centric. Both approaches can extend in two dimensions.
- Extension with new *data* is made possible by abstracting over the data type visitor.
- Individual extensions are again merged using mixin composition.
- Explicit selftypes are needed to pass a visitor along the tree.
- Now, merging two *operation extensions* is easy, requires only a flat mixin composition.
- Merging two *data extensions* is harder, since it requires to merge nested classes as well, using a deep mixin composition.
- So in a sense, we have made the two approaches more compatible, but we have not eliminated their differences.

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## Conclusion

- We have developed two dual families of solutions to the expression problem in Scala.
- New variants: Tree transformers, binary methods (see paper).
- New concern: Independent extensibility.
- Solutions use standard technology (in the Scala world), which shows up in almost every component architecture.
  - abstract types
  - mixin composition
  - explicit selftypes
- This further strengthens the conjecture that the expression problem is indeed a good representative for component architecture in general.

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