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Standard Form

Theorem: Every process is structurally congruent to a standard form.

Proof ?

- make sure that you properly understand the definition of *standard form*
- "induction" on the structure of process terms
- *algebraic reformulation* of terms, using any number of $\equiv$-laws

Natural vs. Structural Induction

Prove that *Prop*(n) is true for $n \in \mathbb{N}$

1. Prove *Prop*(0).
2. Prove that *Prop*$(n+1)$ is true under the condition that *Prop*(n) is true.

Prove that *Prop*(P) is true for $P \in \mathcal{P}$

1. Prove *Prop*$(\mathbf{0})$.
2. (a) Prove that *Prop*$(\mu.P)$ is true under the condition that *Prop*(P) is true.
   (b) Prove that *Prop*$(P|Q)$ $\ldots$

Example: Lottery [Mil99, § 4.5]

$$\text{Lotspec} \stackrel{\text{def}}{=} \tau.b_1.\text{Lotspec} + \cdots + \tau.b_n.\text{Lotspec}$$

$$A(\,a,b,c\,) \stackrel{\text{def}}{=} \overline{a}.C \qquad A_i(\,\vec{a},\vec{b}\,) \stackrel{\text{def}}{=} A\langle\, a_i, b_i, a_{i+1}\,\rangle$$

$$B(\,a,b,c\,) \stackrel{\text{def}}{=} b.C \qquad B_i(\,\vec{a},\vec{b}\,) \stackrel{\text{def}}{=} B\langle\, a_i, b_i, a_{i+1}\,\rangle$$

$$C(\,a,b,c\,) \stackrel{\text{def}}{=} \tau.B + c.A \qquad C_i(\,\vec{a},\vec{b}\,) \stackrel{\text{def}}{=} C\langle\, a_i, b_i, a_{i+1}\,\rangle$$

$$L_1 \stackrel{\text{def}}{=} (\boldsymbol{\nu} a_1 a_2 a_3)(\, C_1 \mid A_2 \mid A_3\,)$$

$$L_2 \stackrel{\text{def}}{=} (\boldsymbol{\nu} a_1 a_2 a_3)(\, A_1 \mid C_2 \mid A_3\,)$$

$$L_3 \stackrel{\text{def}}{=} (\boldsymbol{\nu} a_1 a_2 a_3)(\, A_1 \mid A_2 \mid C_3\,)$$

What is $\text{fn}(L_i)$? Draw the transition graph of L_i !

Stability

<u>Definition:</u>
A process P is called **stable**,
if there is no P' with $P \rightarrow P'$
(abbreviated: $P \not\rightarrow$).

But, reactions are only one half of the story ...

Towards Transition Rules

$$
\begin{aligned}
A &\stackrel{\text{def}}{=} a.A' \\
A' &\stackrel{\text{def}}{=} \overline{b}.A \\
B &\stackrel{\text{def}}{=} b.B' \\
B' &\stackrel{\text{def}}{=} \overline{c}.B
\end{aligned}
$$

What are the transitions of $A|B$?

LTS $(\mathcal{P}, \mathcal{T})$ (I)

Definition: The LTS $(\mathcal{P}, \mathcal{T})$ of concurrent processes over $\mathcal{A} \cup \{\tau\}$ has $\mathcal{P}$ as states, and its transitions $\mathcal{T}$ are generated by the following rules:

$$\text{PRE: } \mu.P \xrightarrow{\mu} P$$

$$\text{SUM}_1\text{: } \frac{M_1 \xrightarrow{\mu} M_1'}{M_1 + M_2 \xrightarrow{\mu} M_1'} \qquad \text{SUM}_2\text{: } \frac{M_2 \xrightarrow{\mu} M_2'}{M_1 + M_2 \xrightarrow{\mu} M_2'}$$

LTS over $(\mathcal{P}, \mathcal{T})$ (II)

PAR$_1$: $$\frac{P_1 \xrightarrow{\mu} P_1'}{P_1|P_2 \xrightarrow{\mu} P_1'|P_2}$$

PAR$_2$: $$\frac{P_2 \xrightarrow{\mu} P_2'}{P_1|P_2 \xrightarrow{\mu} P_1|P_2'}$$

REACT: $$\frac{P \xrightarrow{\lambda} P' \qquad Q \xrightarrow{\overline{\lambda}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

RES: $$\frac{P \xrightarrow{\mu} P'}{(\boldsymbol{\nu} a)\, P \xrightarrow{\mu} (\boldsymbol{\nu} a)\, P'}$$ IF $\mu \notin \{a, \overline{a}\}$

LTS over $(\mathcal{P}, \mathcal{T})$ (III)

$$\text{DEF: } \frac{\{^{\vec{b}}/_{\vec{a}}\}P_A \xrightarrow{\mu} P'}{A\langle\vec{b}\rangle \xrightarrow{\mu} P_A} \text{ IF } A(\vec{a}) \stackrel{\text{def}}{=} P_A$$

$$\text{ALPHA: } \frac{Q \xrightarrow{\mu} Q'}{P \xrightarrow{\mu} P'} \text{ IF } P =_\alpha Q \text{ AND } P' =_\alpha Q'$$

Properties

Proposition: If $P \xrightarrow{\mu} P'$ and $P \equiv Q$, then there is Q' such that $Q \xrightarrow{\mu} Q'$ and $P' \equiv Q'$.

Proposition: Let $P \xrightarrow{\lambda} P'$. Then

$$\begin{aligned} P &\equiv (\boldsymbol{\nu} \vec{z})\,(\,\lambda.Q + M \mid R\,) \\ P' &\equiv (\boldsymbol{\nu} \vec{z})\,(\,Q \mid R\,) \end{aligned}$$

for $\{\lambda, \overline{\lambda}\} \cap \vec{z} = \emptyset$.

Theorem: $P \rightarrow P'$ iff $P \xrightarrow{\tau} \equiv P'$.

Transition Induction (Depth of Infer.)

Prove *Prop*(t) for $t = (P, \alpha, P') \in \mathcal{T}$

1. Prove *Prop*(concl) for all axioms

   $$\textsc{axiom}: \ \text{concl}$$

2. Prove that *Prop*(concl) is true under the condition that *Prop*(prem_i) is true for all premisses, and repeat this for all rules

   $$\textsc{rule}: \ \frac{\text{prem}_1 \qquad \ldots \qquad \text{prem}_n}{\text{concl}}$$

Properties

Proposition:

1. Given P,
   there are finitely many transitions $P \xrightarrow{\mu} P'$.
2. If $P \xrightarrow{\mu} P'$, then $\mathrm{fn}(P', \mu) \subseteq \mathrm{fn}(P')$.
3. If $P \xrightarrow{\mu} P'$ and σ any substitution,
   then $\sigma P \xrightarrow{\sigma\mu} \sigma P'$.

Proof ?

Lottery

- complete the transition graph !
- verify the transitions formally, i.e., using the transition rules.

Bisimilarity on Concurrent Processes

Definition: (* . . . learned by heart . . . *)

Theorem:

1. Structural congruence is a bisimulation.
2. If $P \equiv Q$, then $P \sim Q$.

Homework:

- Do the semaphore example [Milner, 5.14–16].
- Check the proofs of [Milner, §5].

“Algebraic” Properties

- $a \mid b \sim a.b + b.a$
- For all $P \in \mathcal{P}$, $P \sim \sum\{\beta.Q \mid P \xrightarrow{\beta} Q\}$.
- For all $n \geq 0$ and $P_1, \ldots, P_n \in \mathcal{P}$:
  $P_1|\cdots|P_n \sim$
  $$\begin{cases} \sum\{\ \beta.(\ P_1|\cdots|P_i'|\cdots|P_n\ ) \\ \qquad\qquad \mid 1{\leq}i{\leq}n, P_i \xrightarrow{\beta} P_i'\ \} \\ +\sum\{\ \textcolor{red}{\tau}.(\ P_1|\cdots|P_i'|\cdots|P_j'|\cdots|P_n\ ) \\ \qquad\qquad\quad \mid 1{\leq}i{<}j{\leq}n, P_i \xrightarrow{\lambda} P_i', P_j \xrightarrow{\overline{\lambda}} P_j'\ \} \end{cases}$$

“Algebraic” Properties (II)

For all $n \geq 0$, $P_1, \ldots, P_n \in \mathcal{P}$, and $\vec{a}$:

$$
(\boldsymbol{\nu}\vec{a})\,(\,P_1|\cdots|P_n\,) \sim
\begin{cases}
\sum\{\ \beta.(\boldsymbol{\nu}\vec{a})\,(\,P_1|\cdots|P_i'|\cdots|P_n\,) \\
\qquad\qquad |\ 1 \leq i \leq n, P_i \xrightarrow{\beta} P_i', \text{ and } \beta, \overline{\beta} \notin \vec{a}\ \} \\
+ \sum\{\ \tau.(\boldsymbol{\nu}\vec{a})\,(\,P_1|\cdots|P_i'|\cdots|P_j'|\cdots|P_n\,) \\
\qquad\qquad\quad |\ 1 \leq i < j \leq n, P_i \xrightarrow{\lambda} P_i', P_j \xrightarrow{\overline{\lambda}} P_j'\ \}
\end{cases}
$$

Now, recall standard forms !
Expansion **Law** !

“Algebraic” Properties (III)

- $\beta.P + \beta.P + M \sim \beta.P + M$
- $(\boldsymbol{\nu} a)\, a.P \sim \mathbf{0}$
- $(\boldsymbol{\nu} a)\, \overline{a}.P \sim \mathbf{0}$
- $(\boldsymbol{\nu} c)\,(\ a.c.P \mid b.\overline{c}.Q\ ) \sim (\boldsymbol{\nu} c)\,(\ a.c.Q \mid b.\overline{c}.P\ )$
- $\ldots$

Congruence Properties

Proposition:

Bisimilarity is a process congruence, i.e., . . .