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Scope extension . . .

What is the difference between P and P' ?

$$P \;=\; (\boldsymbol{\nu} a)\,(\; a.Q_1 + b.Q_2 \mid \overline{a}.\mathbf{0} \;) \mid \overline{b}.R_1 + \overline{a}.R_2$$

$$P' \;=\; (\boldsymbol{\nu} a')\,(\; a'.Q_1 + b.Q_2 \mid \overline{a}'.\mathbf{0} \mid \overline{b}.R_1 + \overline{a}.R_2 \;)$$

Process Contexts

<u>**Definition:**</u> A **process context** $C[\cdot]$ is (precisely) defined by the following syntax:

$$\begin{array}{rcl} C[\cdot] & ::= & [\cdot] \quad \Big| \quad \alpha.C[\cdot] + M \quad \Big| \quad M + \alpha.C[\cdot] \\ & \Big| & (\boldsymbol{\nu} a)\, C[\cdot] \quad \Big| \quad C[\cdot]|P \quad \Big| \quad P|C[\cdot] \end{array}$$

The **elementary contexts** are $\alpha.[\cdot] + M$, $M + \alpha.[\cdot]$, $(\boldsymbol{\nu} a)\, [\cdot]$, $[\cdot]|P$, $P|[\cdot]$.

$C[Q]$ denotes the result of filling the hole $[\cdot]$ of $C[\cdot]$ with process Q.

Process congruence

Definition: (Process congruence)
Let $\cong$ be an equivalence relation over $\mathcal{P}$.
Then $\cong$ is said to be a process congruence,
if it is preserved by all elementary contexts;
i.e., if $P \cong Q$, then

$$\begin{array}{rclcrcl} \alpha.P + M & \cong & \alpha.Q + M & \qquad & P|R & \cong & Q|R \\ M + \alpha.P & \cong & M + \alpha.Q & & R|P & \cong & R|Q \end{array}$$

$$(\boldsymbol{\nu} a)\, P \;\cong\; (\boldsymbol{\nu} a)\, Q \;.$$

Process congruence (II)

Proposition:

An arbitrary equivalence relation $\cong$ is a process congruence if and only if, for *all* contexts $C[\cdot]$, $P \cong Q$ implies $C[P] \cong C[Q]$.

Note:

For proving that an equivalence relation is a congruence, the elementary contexts suffice.

Structural Congruence

Definition: Structural congruence, written $\equiv$, is the (smallest) process congruence over $\mathcal{P}$ determined by the following equations.

1. $=_\alpha$
2. commutative monoid laws for $(\mathcal{P}, +, \mathbf{0})$
3. commutative monoid laws for $(\mathcal{P}, |, \mathbf{0})$
4. $(\boldsymbol{\nu} a)\,(P|Q) \equiv P|(\boldsymbol{\nu} a)\,Q$, if $a \notin \mathrm{fn}(P)$
   $(\boldsymbol{\nu} a)\,\mathbf{0} \equiv \mathbf{0}$, $(\boldsymbol{\nu} ab)\,P \equiv (\boldsymbol{\nu} ba)\,P$
5. $A\langle \vec{b} \rangle \equiv \{^{\vec{b}}/_{\vec{a}}\}M_A$, if $A(\,\vec{a}\,) \stackrel{\mathrm{def}}{=} M_A$.

Structural Congruence (II)

reflexive-symmetric-transitive context closure (of a set of equations)

$$\frac{}{P = P} \qquad \frac{P = Q}{Q = P} \qquad \frac{P = Q \qquad Q = R}{P = R}$$

$$\frac{P = Q}{C[P] = C[Q]}$$ FOR ARBITRARY "PROCESS CONTEXT" $C[\cdot]$

allows **equational reasoning**, i.e. any number of applications, in either direction, to any subterm

It is Easy to see that . . .

If $P \equiv Q$, then $\mathrm{fn}(P) = \mathrm{fn}(Q)$.

Standard Form

Definition:

A process expression $(\boldsymbol{\nu}\vec{a})\,(\ M_1|\cdots|M_n\ )$,
where each M_i is a non-empty sum,
is said to be in **standard form**.
If $n = 0$ then $M_1|\cdots|M_n$ means $\mathbf{0}$.
If $\vec{a}$ is empty then there is no restriction.

Example:

Let $P = (\boldsymbol{\nu}a)\,(\ A\langle\, a\,\rangle \mid (\boldsymbol{\nu}a)\,a.\mathbf{0}\ )$

with $A(\,a\,) \stackrel{\text{def}}{=} a.(\boldsymbol{\nu}a)\,a.\mathbf{0}$

Is there a standard form $\widehat{P}$ with $\widehat{P} \equiv P$?

Standard Form (II)

Theorem: Every process is structurally congruent to a standard form.

Proof ? **Homework !**

Example: [Milner 99: Exercise 4.10]

If $(\boldsymbol{\nu} a)\,P \equiv P$, then $a \notin \mathrm{fn}(P)$.

In other words, in any standard from, *we can ensure* that the outermost restriction only involves free names in some M_i.

Linking

Let P be a process with *ports* l (left) and r (right), then we may *link* two copies of P through:

$$P \frown Q \stackrel{\text{def}}{=} (\boldsymbol{\nu} m)\,(\ \{^m/_r\}P \mid \{^m/_l\}Q\ )$$

Although often omitted due to laziness, is is clearer to make explicit the parameters of P:

$$\begin{aligned} & P\langle\, l, r\,\rangle \frown Q\langle\, l, r\,\rangle \\ \stackrel{\text{def}}{=}\quad & (\boldsymbol{\nu} m)\,(\quad \{^m/_r\}P\langle\, l, r\,\rangle \ \mid\ \{^m/_l\}Q\langle\, l, r\,\rangle \quad) \\ \equiv\quad & (\boldsymbol{\nu} m)\,(\qquad P\langle\, l, m\,\rangle \ \mid\ \qquad Q\langle\, m, r\,\rangle \quad) \end{aligned}$$

Linking (II)

Prove that:

$$P \frown (Q \frown R) \equiv (P \frown Q) \frown R$$

Reaction

Definition: The reaction relation $\longrightarrow$ over $\mathcal{P}$ is generated by the following rules:

TAU: $\tau.P+M \ \longrightarrow\ P$ REACT: $a.P+M \,|\, \overline{a}.Q+N \ \longrightarrow\ P \,|\, Q$

$$\text{PAR: } \frac{P \ \longrightarrow\ P'}{P|Q \ \longrightarrow\ P'|Q} \qquad \text{RES: } \frac{P \ \longrightarrow\ P'}{(\boldsymbol{\nu} a)\, P \ \longrightarrow\ (\boldsymbol{\nu} a)\, P'}$$

$$\text{STRUCT: } \frac{P \ \longrightarrow\ P'}{Q \ \longrightarrow\ Q'} \text{ IF } P \equiv Q \text{ AND } P' \equiv Q'$$

Reaction (II)

Compare to sequential process expressions !

One “sees” the reactions almost directly when transforming a concurrent process expression into a standard form !

However, one is not obliged to do so :-)

Example Reactions

Recall:

$$
\begin{array}{rcl}
P & = & (\boldsymbol{\nu} a)\,(\; a.Q_1 + b.Q_2 \mid \overline{a}.\mathbf{0} \;) \mid \overline{b}.R_1 + \overline{a}.R_2 \\
P & \longrightarrow & (\boldsymbol{\nu} a)\,(\; Q_1 \mid \mathbf{0} \;) \mid \overline{b}.R_1 + \overline{a}.R_2 \\
P & \longrightarrow & (\boldsymbol{\nu} a)\,(\; Q_2 \mid \overline{a}.\mathbf{0} \;) \mid R_1
\end{array}
$$

Use the reaction rules to derive the above reactions formally.