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Reaction (Example 9.2, [Mil99])

$$
\begin{array}{llcccl}
P & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& (\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) & \mid x(u).\overline{u}\langle v\rangle & \mid \overline{x}\langle z\rangle & \big) \\
P_1 & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& \mathbf{0} & \mid \overline{y}\langle v\rangle & \mid \overline{x}\langle z\rangle & \big) \\
P_2 & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& (\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) & \mid \overline{z}\langle v\rangle & \mid \mathbf{0} & \big) \\
P_3 & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& \overline{v}\langle y\rangle & \mid \mathbf{0} & \mid \mathbf{0} & \big)
\end{array}
$$

Exercise 9.3:
Write down a process Q such that $Q|P_1$ has a redex, but $Q|P_2$ has no redex except that in P_2.

Recursion

$A(\vec{x}) \stackrel{\text{def}}{=} Q_A$, where $Q_A \stackrel{\text{def}}{=} \cdots A\langle \vec{u} \rangle \cdots A\langle \vec{v} \rangle \cdots$
can be used in: $P \stackrel{\text{def}}{=} \cdots A\langle \vec{y} \rangle \cdots A\langle \vec{z} \rangle \cdots$

can be modeled through:

1. invent a to stand for A
2. for any R, let $\widehat{R}$ denote the result of replacing any call $A\langle \vec{w} \rangle$ by $\overline{a}\langle\vec{w}\rangle.\mathbf{0}$
3. replace P by

$$(\boldsymbol{\nu} a)\left(\widehat{P} \mid \mathbf{!}\, a(\vec{x}).\widehat{Q_A} \right)$$

Polyadism

$$[\![\, \overline{y}\langle\vec{z}\rangle.P \,]\!] \stackrel{\text{def}}{=}$$

$$[\![\, y(\vec{x}).P \,]\!] \stackrel{\text{def}}{=}$$

Think about:

$$\overline{y}\langle z_1, z_2\rangle.P \mid y(x_1, x_2, x_3).Q \longrightarrow$$

$$\overline{y}\langle z_1, z_2\rangle.P \mid y(x_1, x_2).Q \mid \overline{y}\langle w_1, w_2\rangle.R \longrightarrow$$

$$[\![\, \overline{y}\langle\vec{z}\rangle.P \,]\!] \stackrel{\text{def}}{=}$$

$$[\![\, y(\vec{x}).P \,]\!] \stackrel{\text{def}}{=}$$

Abstractions

Goal: one single binder!

$$F \quad ::= \quad P \quad \Big| \quad (x).F$$

Abbreviation: $(\vec{x}).P \stackrel{\text{def}}{=} (x_1).\cdots(x_n).P$

Application: $((x).F)\langle\, y \,\rangle \stackrel{\text{def}}{=} \{^{y}\!/_{x}\}F$

"Sugar":

$$x(\vec{y}).P \quad \text{as} \quad x(\vec{y}).P$$

$$(\boldsymbol{\nu}x)\,P \quad \text{as} \quad \boldsymbol{\nu}(x).P$$

$$A(\,\vec{x}\,) \stackrel{\text{def}}{=} Q \quad \text{as} \quad A \stackrel{\text{def}}{=} (\vec{x}).Q$$

Example: Linking

$$
\begin{aligned}
F &\stackrel{\text{def}}{=} (l,r).P \\
G &\stackrel{\text{def}}{=} (l,r).Q
\end{aligned}
$$

$$
\begin{aligned}
& F \frown G \\
\stackrel{\text{def}}{=}\ & (l,r).(\boldsymbol{\nu} m)\,(\ \{^m/_r\}F\langle\, l,r \,\rangle \mid \{^m/_l\}G\langle\, l,r \,\rangle\ ) \\
\equiv\ & (l,r).(\boldsymbol{\nu} m)\,(\ F\langle\, l,m \,\rangle \mid G\langle\, m,r \,\rangle\ ) \\
\equiv\ & (l,r).(\boldsymbol{\nu} m)\,(\ \{^m/_r\}P \mid \{^m/_l\}Q\ )
\end{aligned}
$$

Unbounded Buffers

$$X \frown Y \stackrel{\text{def}}{=} (i,l,o,r).\,(\boldsymbol{\nu} mn)\,(\, X\langle\, i,l,m,n \,\rangle \mid Y\langle\, m,n,o,r \,\rangle\,)$$

$$B \stackrel{\text{def}}{=} (i,l,o,r).\; i(x).(\, C\langle\, x \,\rangle\langle\, i,l,o,r \,\rangle\,)$$

$$C \stackrel{\text{def}}{=} (x).(i,l,o,r).\; \overline{o}\langle x\rangle.B\langle\, i,l,o,r \,\rangle + i(y).(\, \boxed{C\langle\, y \,\rangle \frown C\langle\, x \,\rangle}\langle\, i,l,o,r \,\rangle\,)$$

Labeled Transition Semantics

For inputs:

either $y(\vec{x}).P \xrightarrow{y(\vec{x})} P$

or $y(\vec{x}).P \xrightarrow{y\vec{z}} \{^{\vec{z}}/_{\vec{x}}\}P$ for all $z \in \mathcal{N}$

or just $y(\vec{x}).P \xrightarrow{y} (\vec{x}).P$?

And then for outputs?

either $\overline{y}\langle\vec{z}\rangle.P \xrightarrow{\overline{y}\langle\vec{z}\rangle} P$

or just $\overline{y}\langle\vec{z}\rangle.P \xrightarrow{y} \langle\vec{z}\rangle.P$?

Concretions ("Co-Abstractions")

Concretions are forms $(\boldsymbol{\nu}\vec{y})\,\langle\vec{z}\rangle.P$.
Compare to abstractions $(\vec{x}).P$.

Abstractions and concretions
have **arity** $|\vec{z}| \geq 0$ and $|\vec{x}| \geq 0$, respectively.

An **agent** is an abstraction or a concretion.
Processes are agents; a process is both an abstraction and a concretion (with arity 0).

Structural congruence on agents ...

Language Adaptation/Extension

$$M \quad ::= \quad \mathbf{0} \quad \Big| \quad xF \quad \Big| \quad \bar{x}C \quad \Big| \quad \tau P \quad \Big| \quad M + M$$

$$(\boldsymbol{\nu} z)\, A \quad \stackrel{\text{def}}{=} \quad \ldots$$

$$A \mid Q \quad \stackrel{\text{def}}{=} \quad \ldots$$

Application & Reaction

Application $F@C$ of
an abstraction F and a concretion C
of equal arity is defined as follows:

$$(\vec{x}).P \ @ \ (\boldsymbol{\nu}\vec{z})\,\langle\vec{v}\rangle.Q \ \stackrel{\mathrm{def}}{=} \ (\boldsymbol{\nu}\vec{z})\,(\{^{\vec{v}}/_{\vec{x}}\}P \mid Q)$$

where we assume that $\vec{z}$ are not free in $(\vec{x}).P$.

The rule for reaction becomes:

$$\text{REACT: } \frac{|F| = |C|}{yF{+}M \mid \bar{y}C{+}N \ \rightarrow \ F@C}$$

Commitment (I)

Definition: $\stackrel{\alpha}{\longrightarrow}$ over $\mathcal{P}$ is generated by:

L-SUM: $\alpha A + M \stackrel{\alpha}{\longrightarrow} A$ R-SUM: $M + \alpha A \stackrel{\alpha}{\longrightarrow} A$

$$\text{L-COM:} \quad \frac{P \stackrel{x}{\longrightarrow} F \qquad Q \stackrel{\bar{x}}{\longrightarrow} C}{P \mid Q \stackrel{\tau}{\longrightarrow} F@C}$$

$$\text{R-COM:} \quad \frac{P \stackrel{\bar{x}}{\longrightarrow} C \qquad Q \stackrel{x}{\longrightarrow} F}{P \mid Q \stackrel{\tau}{\longrightarrow} F@C}$$

Commitment (II)

$$\text{L-PAR: } \frac{P \xrightarrow{\alpha} A}{P|Q \rightarrow A|Q} \qquad \text{R-PAR: } \frac{Q \xrightarrow{\alpha} A}{P|Q \rightarrow A|P}$$

$$\text{RES: } \frac{P \xrightarrow{\alpha} A}{(\boldsymbol{\nu} x)\, P \rightarrow (\boldsymbol{\nu} x)\, A} \text{ IF } \alpha \notin \{x, \bar{x}\}$$

$$\text{STRUCT: } \frac{P|!\,P \xrightarrow{\alpha} A}{!\,P \xrightarrow{\alpha} A}$$

Strong Bisimulation

A binary relation over $\mathcal{P}$ is a strong simulation if, whenever $P \; \mathcal{S} \; Q$, if $P \xrightarrow{\alpha} A$ then there exists B such that $Q \xrightarrow{\alpha} B$ and $A \; \mathcal{S} \; B$.

$F \; \mathcal{S} \; G$ means

$C \; \mathcal{S} \; D$ means