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Reaction (Example 9.2, [Mil99])

$$
\begin{array}{llcccc}
P & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& (\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) & \mid\ x(u).\overline{u}\langle v\rangle & \mid\ \overline{x}\langle z\rangle & \big) \\
P_1 & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& \mathbf{0} & \mid\ \overline{y}\langle v\rangle & \mid\ \overline{x}\langle z\rangle & \big) \\
P_2 & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& (\overline{x}\langle y\rangle + z(w).\overline{w}\langle y\rangle) & \mid\ \overline{z}\langle v\rangle & \mid\ \mathbf{0} & \big) \\
P_3 & \stackrel{\text{def}}{=} (\boldsymbol{\nu} z)\big(& \overline{v}\langle y\rangle & \mid\ \mathbf{0} & \mid\ \mathbf{0} & \big)
\end{array}
$$

Exercise 9.3:
Write down a process Q such that $Q|P_1$ has a redex, but $Q|P_2$ has no redex except that in P_2.

Recursion

$A(\,\vec{x}\,) \stackrel{\text{def}}{=} Q_A$, where $Q_A \stackrel{\text{def}}{=} \cdots A\langle\,\vec{u}\,\rangle \cdots A\langle\,\vec{v}\,\rangle \cdots$
can be used in: $P \stackrel{\text{def}}{=} \cdots A\langle\,\vec{y}\,\rangle \cdots A\langle\,\vec{z}\,\rangle \cdots$

can be modeled through:

1. invent a to stand for A
2. for any R, let $\widehat{R}$ denote the result of replacing any call $A\langle\,\vec{w}\,\rangle$ by $\overline{a}\langle\vec{w}\rangle.\mathbf{0}$
3. replace P by

$$(\boldsymbol{\nu} a)\,(\ \widehat{P} \mid \mathbf{!}\, a(\vec{x}).\widehat{Q_A}\ )$$

Polyadism

$[\![\,\overline{y}\langle\vec{z}\rangle.P\,]\!] \stackrel{\text{def}}{=}$

$[\![\,y(\vec{x}).P\,]\!] \stackrel{\text{def}}{=}$

Think about:

$$\overline{y}\langle z_1, z_2\rangle.P \mid y(x_1, x_2, x_3).Q \;\longrightarrow$$

$$\overline{y}\langle z_1, z_2\rangle.P \mid y(x_1, x_2).Q \mid \overline{y}\langle w_1, w_2\rangle.R \;\longrightarrow$$

$[\![\,\overline{y}\langle\vec{z}\rangle.P\,]\!] \stackrel{\text{def}}{=}$

$[\![\,y(\vec{x}).P\,]\!] \stackrel{\text{def}}{=}$

Abstractions

Goal: one single binder!

$$F \quad ::= \quad P \quad \Big| \quad (x).F$$

Abbreviation: $(\vec{x}).P \stackrel{\text{def}}{=} (x_1).\cdots(x_n).P$

Application: $((x).F)\langle\, y\,\rangle \stackrel{\text{def}}{=} \{{}^{y}\!/\!_{x}\}F$

"Sugar": $x(\vec{y}).P$ as $x(\vec{y}).P$

$(\boldsymbol{\nu} x)\, P$ as $\boldsymbol{\nu}(x).P$

$A(\,\vec{x}\,) \stackrel{\text{def}}{=} Q$ as $A \stackrel{\text{def}}{=} (\vec{x}).Q$

Example: Linking

$$\begin{aligned} F &\stackrel{\text{def}}{=} (l,r).P \\ G &\stackrel{\text{def}}{=} (l,r).Q \end{aligned}$$

$$\begin{aligned} &F \frown G \\ &\stackrel{\text{def}}{=} (l,r).(\boldsymbol{\nu} m)\,(\ \{{}^{m}\!/\!_{r}\}F\langle\, l,r\,\rangle \ \mid\ \{{}^{m}\!/\!_{l}\}G\langle\, l,r\,\rangle\ ) \\ &\equiv (l,r).(\boldsymbol{\nu} m)\,(\quad F\langle\, l,m\,\rangle \ \mid\ G\langle\, m,r\,\rangle\ ) \\ &\equiv (l,r).(\boldsymbol{\nu} m)\,(\quad \{{}^{m}\!/\!_{r}\}P \ \mid\ \{{}^{m}\!/\!_{l}\}Q\ ) \end{aligned}$$

Unbounded Buffers

$$X \frown Y \stackrel{\text{def}}{=} (i,l,o,r).\ (\boldsymbol{\nu} mn)\,(\, X\langle\, i,l,m,n\,\rangle \mid Y\langle\, m,n,o,r\,\rangle\,)$$

$$B \stackrel{\text{def}}{=} (i,l,o,r).\ i(x).(\, C\langle\, x\,\rangle\langle\, i,l,o,r\,\rangle\,)$$

$$C \stackrel{\text{def}}{=} (x).(i,l,o,r).\ \overline{o}\langle x\rangle.B\langle\, i,l,o,r\,\rangle + i(y).(\,\boxed{C\langle\, y\,\rangle \frown C\langle\, x\,\rangle}\langle\, i,l,o,r\,\rangle\,)$$

Labeled Transition Semantics

For inputs:

either $y(\vec{x}).P \xrightarrow{y(\vec{x})} P$

or $y(\vec{x}).P \xrightarrow{y\vec{z}} \{{}^{\vec{z}}\!/\!_{\vec{x}}\}P$ for all $z \in \mathcal{N}$

or just $y(\vec{x}).P \xrightarrow{y} (\vec{x}).P$?

And then for outputs?

either $\overline{y}\langle\vec{z}\rangle.P \xrightarrow{\overline{y}\langle\vec{z}\rangle} P$

or just $\overline{y}\langle\vec{z}\rangle.P \xrightarrow{y} \langle\vec{z}\rangle.P$?

Concretions (“Co-Abstractions”)

Concretions are forms $(\boldsymbol{\nu}\vec{y})\,\langle\vec{z}\rangle.P$.
Compare to abstractions $(\vec{x}).P$.

Abstractions and concretions
have **arity** $|\vec{z}| \geq 0$ and $|\vec{x}| \geq 0$, respectively.

An **agent** is an abstraction or a concretion.
Processes are agents; a process is both an abstraction and a concretion (with arity 0).

Structural congruence on agents ...

Language Adaptation/Extension

$$M \quad ::= \quad \mathbf{0} \quad \Big| \quad xF \quad \Big| \quad \bar{x}C \quad \Big| \quad \tau P \quad \Big| \quad M+M$$

$$(\boldsymbol{\nu}z)\,A \quad \stackrel{\text{def}}{=} \quad \ldots$$

$$A \mid Q \quad \stackrel{\text{def}}{=} \quad \ldots$$

Application & Reaction

Application $F@C$ of
an abstraction F and a concretion C
of equal arity is defined as follows:

$$(\vec{x}).P \; @ \; (\boldsymbol{\nu}\vec{z})\,\langle\vec{v}\rangle.Q \;\stackrel{\text{def}}{=}\; (\boldsymbol{\nu}\vec{z})\,(\{\vec{v}/\vec{x}\}P \mid Q)$$

where we assume that $\vec{z}$ are not free in $(\vec{x}).P$.

The rule for reaction becomes:

$$\text{REACT: } \frac{|F| = |C|}{yF{+}M \mid \bar{y}C{+}N \;\longrightarrow\; F@C}$$

Commitment (I)

<u>Definition:</u> $\stackrel{\alpha}{\longrightarrow}$ over $\mathcal{P}$ is generated by:

$$\text{L-SUM: } \alpha A + M \stackrel{\alpha}{\longrightarrow} A \qquad \text{R-SUM: } M + \alpha A \stackrel{\alpha}{\longrightarrow} A$$

$$\text{L-COM: } \frac{P \stackrel{x}{\longrightarrow} F \qquad Q \stackrel{\bar{x}}{\longrightarrow} C}{P \mid Q \stackrel{\tau}{\longrightarrow} F@C}$$

$$\text{R-COM: } \frac{P \stackrel{\bar{x}}{\longrightarrow} C \qquad Q \stackrel{x}{\longrightarrow} F}{P \mid Q \stackrel{\tau}{\longrightarrow} F@C}$$

Commitment (II)

$$\text{L-PAR: } \frac{P \xrightarrow{\alpha} A}{P|Q \rightarrow A|Q} \qquad \text{R-PAR: } \frac{Q \xrightarrow{\alpha} A}{P|Q \rightarrow A|P}$$

$$\text{RES: } \frac{P \xrightarrow{\alpha} A}{(\boldsymbol{\nu} x)\, P \rightarrow (\boldsymbol{\nu} x)\, A} \text{ IF } \alpha \notin \{x, \bar{x}\}$$

$$\text{STRUCT: } \frac{P|!\,P \xrightarrow{\alpha} A}{!\,P \xrightarrow{\alpha} A}$$

Strong Bisimulation

A binary relation over $\mathcal{P}$ is a strong simulation if, whenever $P \mathrel{\mathcal{S}} Q$, if $P \xrightarrow{\alpha} A$ then there exists B such that $Q \xrightarrow{\alpha} B$ and $A \mathrel{\mathcal{S}} B$.

$F \mathrel{\mathcal{S}} G$ means

$C \mathrel{\mathcal{S}} D$ means