# Part 4: Functional Nets and Join Calculus

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## What's a Functional Net?

- Functional nets arise out of a fusion of key ideas of functional programming and Petri nets.
- Functional programming: Rewrite-based semantics with function application as the fundamental computation step.
- Petri nets: Synchronization by waiting until all of a given set of inputs is present, where in our case

  input = function application.
- A functional net is a concurrent, higher-order functional program with a Petri-net style synchronization mechanism.
- Theoretical foundation: Join calculus.

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## Thesis of this Talk

Functional nets are a simple, intuitive model

of { imperative, functional, concurrent } programming.

Functional nets combine well with OOP.

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## Elements

Functional nets have as elements:

- functions
- objects
- parallel composition

They are presented here as a calculus and as a programming notation.

Calculus: (Object-based) join calculus

Notation: Funnel (alternatives are Join or JoCAML)

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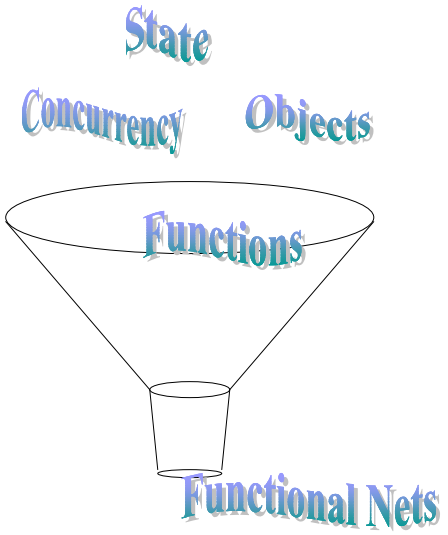
## The Principle of a Funnel

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## Stage 1: Functions

- A simple function definition:

```
def gcd (x, y) =
  if (y == 0) x
  else gcd (y, x % y)
```

- Function definitions start with **def**.
- Operators as in C/Java.
- Usage:

```
val x = gcd (a, b)
print (x * x)
```

- Call-by-value: Function arguments and right-hand sides of **val** definitions are always evaluated.

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## Stage 2: Objects

- One often groups functions to form a single value. Example:

```
def makeRat (x, y) = {
   val g = gcd (x, y)
   {  def numer  =  x / g
      def denom  =  y / g
      def add r  =  makeRat (
                       numer * r.denom + r.numer * denom,
                       denom * r.denom)
      ...
   }
}
```

- This defines a record with functions numer, denom, add, ...
- We identify: Record = Object, Function = Method
- For convenience, we admit parameterless functions such as numer.

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## Functions + Objects Give Algebraic Types

- Functions + Records can encode algebraic types
  - Church Encoding
  - Visitor Pattern
- Example: Lists are represented as records with a single method, match.
- match takes as parameter a *visitor* record with two functions:

```
{ def Nil = ...
  def Cons (x, xs) = ... }
```

- match invokes the Nil method of its visitor if the List is empty, the Cons method if it is nonempty.

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## Lists

- Here is an example how match is used.

```
def append (xs, ys) =
   xs.match {
      def Nil = ys
      def Cons (x, xs1) = List.Cons (x, append (xs1, ys))
   }
```

- It remains to explain how lists are constructed.

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## Lists

- Here is an example how match is used.

```
def append (xs, ys) =
   xs.match {
      def Nil = ys
      def Cons (x, xs1) = List.Cons (x, append (xs1, ys))
   }
```

- It remains to explain how lists are constructed.
- We wrap definitions for Nil and Cons constructors in a List "module". They each have the appropriate implementation of match.

```
val List = {
   def Nil          =   { def match v = ??? }
   def Cons (x, xs) =   { def match v = ??? }
}
```

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## Lists

- Here is an example how match is used.

```
def append (xs, ys) =
   xs.match {
      def Nil = ys
      def Cons (x, xs1) = List.Cons (x, append (xs1, ys))
   }
```

- It remains to explain how lists are constructed.
- We wrap definitions for Nil and Cons constructors in a List "module". They each have the appropriate implementation of match.

```
val List = {
   def Nil          =   { def match v = v.Nil }
   def Cons (x, xs) =   { def match v = v.Cons (x, xs) }
}
```

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## Stage 3: Concurrency

- Principle :
  - Function calls model *events*.
  - & means *conjunction* of events.
  - = means left-to-right rewriting.
  - & can appear on the right hand side of a = (*fork*) as well as on the left hand side (*join*).
- Analogy to Petri-Nets :

| | | |
|---|---|---|
| call | ≈ | place |
| equation | ≈ | transition |

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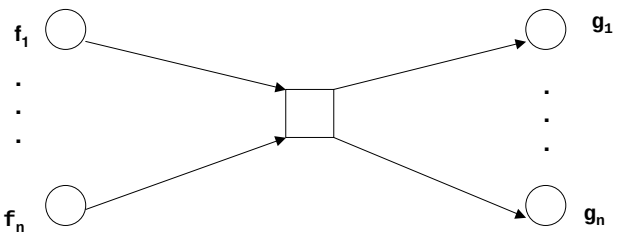
$f_1 \& \ldots \& f = g_1 \& \ldots \& g_n$

corresponds to




- Functional Nets are more powerful:
  - parameters,
  - nested definitions,
  - higher order.

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## Example : One-Place Buffer

Functions : put, get (external)
empty, full (internal)

Definitions :

```
def put x & empty  =  () & full x
    get  & full x  =  x  & empty
```

Usage :

```
val x = get ; put (sqrt x)
```

- An equation can now define more than one function.
- Exercise: Write a Petri net modelling a one-place buffer.

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## Function Results

- In the rewrite rules for a one place buffer we still have to specify to which function call a result should be returned.
- Principle: In a rewrite rule wich joins n functions

  $f_1$ & ... & $f_n$ = E

  the result of E (if there is one) is returned to the first function $f_1$. All other functions do not return a result.
- We call functions which return a result *synchronous* and functions which don't *asynchronous*.
- It's also possible to have rewrite rules with only asynchronous functions. Example:

```
def double & g x = g x & g x
```

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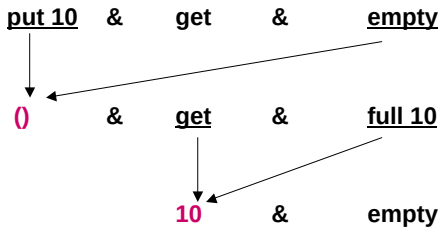
## Rewriting Semantics

- A set of calls which matches the left-hand side of an equation is replaced by the equation 's right-hand side (after formal parameters are replaced by actual parameters).
- Calls which do not match a left-hand side block until they form part of a set which does match.
- Example:

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## Objects and Joins

- We'd like to make a constructor function for one-place buffers.
- We could use tuples of methods:

```
def newBuffer = {
   def put x &  empty   =    () &  full x,
       get   &  full x    =    x  &  empty
   (put, get)  &  empty
}
val (bput, bget) = newBuffer ; ...
```

- But this quickly becomes combersome as number of methods grows.
- Usual record formation syntax is also not suitable
  - we need to hide function symbols
  - we need to call some functions as part of initialization.

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## Qualified Definitions

- Idea: Use qualified definitions:

```
def newBuffer = {
  def this.put x &  empty  =   ()  &  full x,
      this.get   &  full x   =   x   &  empty
  this  &  empty
}
val buf  =  newBuffer ; ...
```

- Three names are defined in the local definition:

| | |
|---|---|
| this | - a record with two fields, get and put. |
| empty | - a function |
| full | - a function |

- this is returned as result from newBuffer; empty and full are hidden.

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- The choice of this as the name of the record was arbitrary; any other name would have done as well.
- We retain a conventional record definition syntax as an abbreviation, by inserting implicit prefixes. E.g.

```
{ def  numer  =  x / g
  def  denom =  y / g }
```

is equivalent to

```
{ def  r.numer = x / g, r.denom = y / g ; r }
```

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## Mutable State

- A *variable* (or reference cell) with functions

  read, write (external)
  state (internal)

  is created by the following function:

```
def newRef init = {
   def this.read  &  state x  =  x  &  state x,
       this.write y &  state x  =  () &  state y
   this & state init
}
```

- Usage:

```
val r = newRef 0 ; r.write (r.read + 1)
```

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## Control Structures

- Imperative control structures can be formulated as higher order functions.
- Example: while loop

```
while (cond) (body)  =
   if (cond ()) {
      body () ; while (cond) (body)
   } else {
      ()
   }
```

- Usage:

```
while (| i < N & !found) (| found := f (i) ; i := next (i))
```

- Exercise: Write functions that implement repeat and for loops.

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## Stateful Objects

- An object with *methods* $m_1,...,m_n$ and *instance variables* $x_1,...,x_k$ can be expressed such :

$$\begin{aligned}
&\textbf{def}\ \ this.m_1\ \&\ state\ (x_1,...,x_k)\ =\ ...\ ;\ state\ (y_1,...,y_k),\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad :\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad :\\
&\qquad this.m_n\ \&\ state\ (x_1,...,x_k)\ =\ ...\ ;\ state\ (z_1,...,z_k);\\
&\\
&this\ \ \&\ \ \ state\ (init_1,...,\ init_k)
\end{aligned}$$

« Result » « initial state »

- The encoding enforces *mutual exclusion*, makes the object into a *monitor.*

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## Object Identity

- One often characterizes objects as having "state, behavior and identity".
- We model state with instance variables and behavior with methods, but what about identity?
- Question: Can we define an operation == such that for objects X, Y, X == Y is true iff X and Y are the same object (i.e. have been created by the same operation)?
- Need cooperation of the object.

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## Objects with Identity

- We want to define a method eq with one parameter, so that A == B can be implemented as A.eq(B).
- Idea: Make use of a boolean instance variable which is normally set to false. Then eq can be implemented by setting the variable to true and testing whether the *other* object's variable is also true.

```
def newObjectWithIdentity = {
   def this.eq other & flag x =  resetFlag (other.testFlag & flag true)
       this.testFlag  & flag x =  x & flag x
       resetFlag y    & flag x =  y & flag false

       ... (other definitions) ...

   this  &  flag false
}
```

- Does this work in a setting where several threads run concurrently?

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## Synchronization

- Functional nets are very good at expressing many process synchronization techniques.
- Example: A semaphore (or: lock) offers two operations, getLock and releaseLock, which bracket a region which should be executed atomically.
- The getLock operation blocks until the lock is available. The releaseLock operation is asynchronous.
- This is implemented as follows:

```
def newLock = {
   def this.getLock & this.releaseLock = ()
   this & ths.releaseLock
}
```

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## Using Semaphores

- Semaphores can be used as follows:

```
lock = newLock
client1 = {
   ...
   lock.getLock ; ... /* critical region */ ... ; lock.releaseLock
   ...
}
client2 = {
   ...
   lock.getLock ; ... /* critical region */ ... ; lock.releaseLock
   ...
}
client1 & client2
```

- Problem: It's easy to forget a getLock or releaseLock operation ina client. Can you design a solution which passes a critical region to a single higher order function, sync?

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## Monitors

- A monitor is an object in which only one method can execute at any one time.
- This is easy to model as a functional net: Simply add an asynchronous function turn, which is consumed at each call and which is re-called after a method has executed:

```
def f  & turn = ... ; turn
    g  & turn = ... ; turn
```

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## Exercise: Bounded Buffer

- Let's implement a bounded buffer as a function net.
- Without taking overflow/underflow or concurrency into account, such a buffer could be written as follows:

```
def newBuffer (N) = {
   val elems = Array.new (N)
   var in := 0; var out := 0;
   def put (x) = {
      elems.put (in, x) ; in := (in + 1) % N ;
   }
   def get = {
      val x = elems.get (out) ; out := (out + 1) % N
      x
   }
}
```

- The parameter N indicates the buffer's size.

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- This assumes arrays which are created with

```
Array.new
```

and which offer operations:

```
get (index)
put (index, value)
```

- Question: How can we modify newBuffer, so that
  - a buffer can be accessed by several processes running concurrently
  - A put operation blocks as long as the buffer is full.
  - A get operation blocks as long as the buffer is empty.

?

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```
def newBuffer (N) = {
   val elems = Array.new (N)
   var in := 0; var out := 0; var n := 0
   def put (x)
      elems.put (in, x) ; in := (in + 1) % N

   }
   def get
      val x = elems.get (out) ; out := (out + 1) % N
      x
    }


}
```

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## Readers/Writers Synchronization.

- Readers/writers is a more refined synchronization technique.
- Specification: Implement operations startRead, startWrite, endRead, endWrite such that:
  - there can be multiple concurrent reads,
  - there can be only one write at one time,
  - reads and writes are mutually exclusive,
  - pending write requests have priority over pending reads, but don 't preempt ongoing reads.

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## First Version

- Introduce two auxiliary state functions

  readers n - the number of *active* reads

  writers n - the number of *pending* writes

- Equations:

```
def startRead  & writers 0   =  startRead1,
    startRead1 & readers n   =  () & writers 0 & readers (n+1),

    startWrite  & writers n  =  startWrite1 & writers (n+1),
    startWrite1 & readers 0  =  (),

    endRead  & readers n     =  readers (n-1),
    endWrite & writers n     =  writers (n-1) & readers 0

readers 0 & writers 0
```

- Note the almost-symmetry between startRead and startWrite, which reflects the different priorities of readers and writers.

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## Final program

- The previous program was is not yet legal Funnel since it contained numeric patterns.
- We can get rid of value patterns by partitioning state functions.

```
def startRead  & noWriters   =  startRead1,
    startRead1 & noReaders   =  () & noWriters & readers 1,
    startRead1 & readers n   =  () & noWriters & readers (n+1),

    startWrite  & noWriters  =  startWrite1 & writers 1,
    startWrite  & writers n  =  startWrite1 & writers (n+1),
    startWrite1 & noReaders  =  (),

    endRead  & readers n     =  if (n == 1) noReaders else (readers (n-1)),
    endWrite & writers n     =  noReaders &
                                ( if (n == 1) noWriters else writers (n-1) )
noWriters & noReaders
```

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A packaged readers/writers synchronization structure is then written as follows:

```
def newReadersWriters = {
  def this.startRead   & noWriters =   startRead1,
      startRead1 & noReaders    =   () & noWriters & readers 1,
      startRead1 & readers n    =   () & noWriters & readers (n+1),

      this.startWrite  & noWriters =  startWrite1 & writers 1,
      this.startWrite  & writers n =  startWrite1 & writers (n+1),
      startWrite1 & noReaders    =   (),

      this.endRead     & readers n =  if (n == 1) noReaders else (readers (n-1)),
      this.endWrite    & writers n =  noReaders &
                                      ( if (n == 1) noWriters else writers (n-1) )
  this & noWriters & noReaders
}
```

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## Summary : Concurrency

- Functional nets support an event-based model of concurrency.
- Channel based formalisms such as CCS, CSP or π - Calculus can be easily encoded.
- High-level synchronization à la Petri-nets.
- Takes work to map to instructions of hardware machines.
- Options:
  - Search patterns linearly for a matching one,
  - Construct finite state machine that recognizes patterns,
  - others?

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## Message Passing

- Process algebras often use *message passing* as the fundamental communication primitive.
- Example: Pi-Calculus
  - One data type: the channel.
  - Channels support read and write operations.
  - Reads on a channel block until some data is written to the channel.
  - Two variants:
    *synchronous* (a write blocks until data is read)
    *asynchronous* (writes don't block)

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## Asynchronous Channels

- Asynchronous channels can be implemented in Funnel as follows:

```
def newAsyncChannel = {
  def this.read & this.write x = x
  this
}
```

- Note similarity to semaphore implementation.
- Typical usage scenario:

```
def c = newAsyncChannel
def producer = { while (|alive) (| ... c.write x ... ) }
def consumer = { while (|alive) (| ... val x = c.read ... ) }
```

- Problems: Unlimited pile-up of undelivered messages, message ordering.

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## Foundations

- We now develop a formal model of functional nets.
- The model is based on an adaptation of join calculus (Fournet & Gonthier 96)
- Two stages: sequential, concurrent.

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## A Calculus for Functions and Objects

- Name-passing, continuation passing calculus.
- Closely resembles intermediate language of FPL compilers.

Syntax:

| | | | |
|---|---|---|---|
| Names | x, y, z | | |
| Identifiers | i, j, k | ::= | x \| i.x |
| Terms | M, N | ::= | i j \| **def** D ; M |
| Definitions | D | ::= | L = M \| D, D \| 0 |
| Left-hand Sides | L | ::= | i x |

Reduction:

**def** D, i x = M ; ... i j ... → **def** D, i x = M ; ... [j/x] M ...

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## A Calculus for Functions and Objects

- The ... ... dots are made precise by a *reduction context*.
- Same as Felleisen's evaluation contexts but there's no evaluation here.

Syntax:

| | | | |
|---|---|---|---|
| Names | x, y, z | | |
| Identifiers | i, j, k | ::= | x \| i.x |
| Terms | M, N | ::= | i j \| **def** D ; M |
| Definitions | D | ::= | L = M \| D, D \| 0 |
| Left-hand Sides | L | ::= | i x |
| Reduction Contexts | R | ::= | [ ] \| **def** D ; R |

Reduction:

**def** D, i x = M ; R[ i j ] → **def** D, i x = M ; R[ [j/x]M ]

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## Structural Equivalence

- Alpha renaming: Local names may be consistently renamed as long as this does not introduce variable clashes.
- Comma is associative and commutative, with the empty definition 0 as identity

| | | |
|---|---|---|
| $D_1, D_2$ | ≡ | $D_2, D_1$ |
| $D_1, (D_2, D_3)$ | ≡ | $(D_1, D_2), D_3$ |
| 0, D | ≡ | D |

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## Properties

- Name-passing calculus - every value is a (qualified) name.
- Contrast to lambda calculus, where values are lambda abstractions.
- Mutually recursive definitions are built in.
- Functions with results and value definitions are encoded via a CPS transform (see later).
- Tuples can be encoded:

f (i, j) ≡ ( **def** ij.fst () = i, ij.snd () = j ; f ij )

f (x, y) = M ≡ f xy = ( **val** x = xy.fst () ; **val** y = xy.snd () ; M )

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## A Calculus for Functions, Objects and Concurrency

Syntax:

| | | | |
|---|---|---|---|
| Names | x, y, z | | |
| Identifiers | i, j, k | ::= | x \| i.x |
| Terms | M, N | ::= | i j \| **def** D ; M \| M & M |
| Definitions | D | ::= | L = M \| D, D \| 0 |
| Left-hand Sides | L | ::= | i x \| L & L |
| Reduction Contexts | R | ::= | [ ] \| **def** D ; R \| R & M \| M & R |

Reduction:

**def** D, $i_1$ $x_1$ & ... & $i_n$ $x_n$ = M ; R [$i_1$ $j_1$ & ... & $i_n$ $j_n$]

→ **def** D, $i_1$ $x_1$ & ... & $i_n$ $x_n$ = M ; R [[$j_1$/$x_1$,...$j_n$/$x_n$] M]

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## Structural Equivalence

- Alpha renaming
- Comma is AC, with the empty definition 0 as identity:
- & is AC:

$M_1, M_2 \equiv M_2, M_1$

$M_1, (M_2, M_3) \equiv (M_1, M_2), M_3$

- Scope Extrusion:

(**def** D ; M) & N ≡ **def** D ; M & N

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## Relation to Join Calculus

- Strong connections to join calculus.
  - \- Polyadic functions
  - \+ Records, via qualified definitions and accesses.
- Formulated here as a rewrite system, whereas original join uses a reflexive CHAM.
- The two formulations are equivalent.

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## Continuation Passing Style

- Note that there is no term form which can represent a value. Hence, nothing can ever be returned from a join calculus expression.
- Instead, every "value-returning" function f is passed another function k as a parameter. k is called a *continuation* for f. The result of f is passed as a parameter to k.
- That is, instead of

```
def f () = 1 ; ... print (f ())
```

  one writes

```
def f (k) = k 1 ; ... f (print)
```

- This is called *continuation passing style* (in contrast to *direct style*).

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## One-Place Buffer in Continuation Passing Style

- Here is the one-place buffer in continuation passing style

```
def newBuffer k1 = (
   def this.put (x, k2) & empty   =   k2 ()  & full x ,
       this.get k3      & full x   =   k3 x   & empty ;
   k1 this  &   empty
)
```

- This formulation fits our syntax for object-based join calculus.
- Note that only functions which were synchronous in direct style get continuation parameters; asynchronous functions stay as they were.
- In a sense, the continuation argument represents a function's return address.

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## From Direct to Continuation Passing Style

- Is it possible to map from direct style to continuation passing style?
- This is the task of a *continuation passing transform*.
- The transform takes programs written in direct style and maps them into equivalent programs written in continuation passing style.

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## Direct Style Funnel

- The target of the CP transform is object-based join calculus.
- Its source is the same in direct style, with the additional term constructs:

```
E  ::=  ...
     |  I                    result
     |   val x = E ; E'       value definition
```

- The transform can not be expressed as a simple macro expansion.
- Instead, we need to carry along an additional parameter k, which represents the continuation function to which the result of the translated term should be passed.
- Schema: TE [ M ] k = ...

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## Continuation Passing Transform

- The CP Transform is defined by three recursive function TE over expressions, TD over definitions, and TL over left-hand sides:

TE [ i] k = k(i)

TE [ **val** x = E ; E'] k = **def** k'(x) = TE [ E' ]; TE [ E ] k'
where k' fresh

TE [ j(i) ] k = j (i, k ) if j is synchronous
= j (i) otherwise

TE [E & E' ] k = TE [ E ] k & TE [ E' ] ⊥

TE [ **def** D ; E ] k = **def** TD [ D ] ; TE [ E ] k

- We require that ⊥ does not appear in translated expressions.

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- On definitions, the CP transform is defined as follows.

TD [ L = E ] = TE [ L ] k = TE [ E ] k where k fresh.

TD [ D, D' ] = TD [ D ] , TD [ D' ]

TD [ 0 ] = 0

- On left-hand sides, the CP transform is defined as follows.

TE [ j (x) ] k = j (x, k ) if j is synchronous
= j (x) otherwise

TE [ L & L' ] k = TE [ L ] k & TE [ L' ] ⊥

- This assumes we know which names are synchronous functions, and which are asynchronous (one way to determine this is by a type system).

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## Nested Applications

- Nested function applications can be added and expanded by means of value definitions:

TD [ F (E) ] k = TD [ val f = F ; f (E) ] k if F is not an identifier

TD [ f (E) ] k = TD [ val x = E; f (x) ] k if E is not an identifier

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## Example

TD [ **def** twice (f, x) = f (f x) ; twice (g, y) ] k

=

TD [ **def** twice (f, x) = ( **val** x' = f x ; f x' ) ; twice (g, y) ] k

=

?

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## Example

```
    TD [ def twice (f, x) = f (f x) ; twice (g, y) ] k
=
    TD [ def twice (f, x) = ( val x' = f x ; f x' ) ; twice (g, y) ] k
=
    def twice (f, x, k1) = (
      def k2 x' = f (x', k1) ; f (x, k2)
    ) ;
    twice (g, y, k)
```

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## Evaluation Order

- The continuation passing transform fixes evaluation order by making every evaluation step explicit. Example:

  TE [ **val** x = E ; E' ] k = **def** k' x = TE [ E' ]; TE [ E ] k'
  where k' fresh

- E is definitely evaluated before E' (call-by-value).
- If we want call-by-name evaluation instead, we can do this by using a different continuation passing transform.

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## Conclusions

- Functional nets provide a simple, intuitive way to think about functional, imperative, and concurrent programs.
- They are based on join calculus.
- Mix-and-match approach: functions (+objects) (+concurrency).
- Close connections to
  - sequential FP (a subset),
  - Petri-nets (another subset),
  - π-Calculus (can be encoded easily).
- Functional nets admit a simple expression of object-oriented concepts.

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