Foundations of Programming

– Concurrency –

Session 13 – April 29, 2002

Uwe Nestmann

EPFL-LAMP

Goals

- Session 13
  - encodings in π
  - towards implementation: asynchrony
  - towards distribution: from π to join
  - from λ to join ?
  - encodings between π & join
- Session 14
  - back to funnel / functional nets

Encoding Tuples

$$\llbracket\, \overline{y}\langle\vec{z}\rangle.P \,\rrbracket \stackrel{\text{def}}{=}$$
$$\llbracket\, y(\vec{x}).P \,\rrbracket \stackrel{\text{def}}{=}$$

Think about:

$$\overline{y}\langle z_1, z_2\rangle.P \mid y(x_1, x_2, x_3).Q \;\longrightarrow$$
$$\overline{y}\langle z_1, z_2\rangle.P \mid y(x_1, x_2).Q \mid \overline{y}\langle w_1, w_2\rangle.R \;\longrightarrow$$

$$\llbracket\, \overline{y}\langle\vec{z}\rangle.P \,\rrbracket \stackrel{\text{def}}{=}$$
$$\llbracket\, y(\vec{x}).P \,\rrbracket \stackrel{\text{def}}{=}$$

Implementing the Pi-Calculus

- goal: design of a programming language on top of π (just like functional languages on top of λ)
- observation: certain constructs are both
  - difficult to implement
  - misinterpretable for the purpose of verification
  - replacable by more primitive notions
- result: a simpler, but (almost) equally expressive pi-calculus
  - only asynchronous output
  - no choice/summation

The Asynchronous Pi-Calculus

P, Q	$::=$	$(\nu y)\, P$	$\mid$	$\overline{y}\langle \tilde{z} \rangle$	$\mid$	$y(\tilde{x}).P$	$\mid$	$P \mid Q$	$\mid$	$*P$
		restriction		output		input		parallel		replication

+ large amount of theory & techniques
  equivalences, (sub-) types, polymorphism, tools . . .
+ enormous expressive power
  functions, ADTs, objects, classes, constraints . . .
+ efficient, type-safe implementation **(Nomadic) Pict**
  [Pierce, Turner ’93–’97], [Wojciechowski ’98–’02]

– **Pict** on monoprocessor: only quasi-parallel

Encoding Synchrony

$$[\![P_1 \mid P_2]\!] \stackrel{\text{def}}{=} [\![P_1]\!] \mid [\![P_2]\!]$$

$$\vdots$$

$$[\![\overline{y}\langle z\rangle.P]\!] \stackrel{\text{def}}{=}$$

$$[\![y(x).P]\!] \stackrel{\text{def}}{=}$$

Encoding Summation

- (only shown on demand)

Encoding Lambda-Calculus

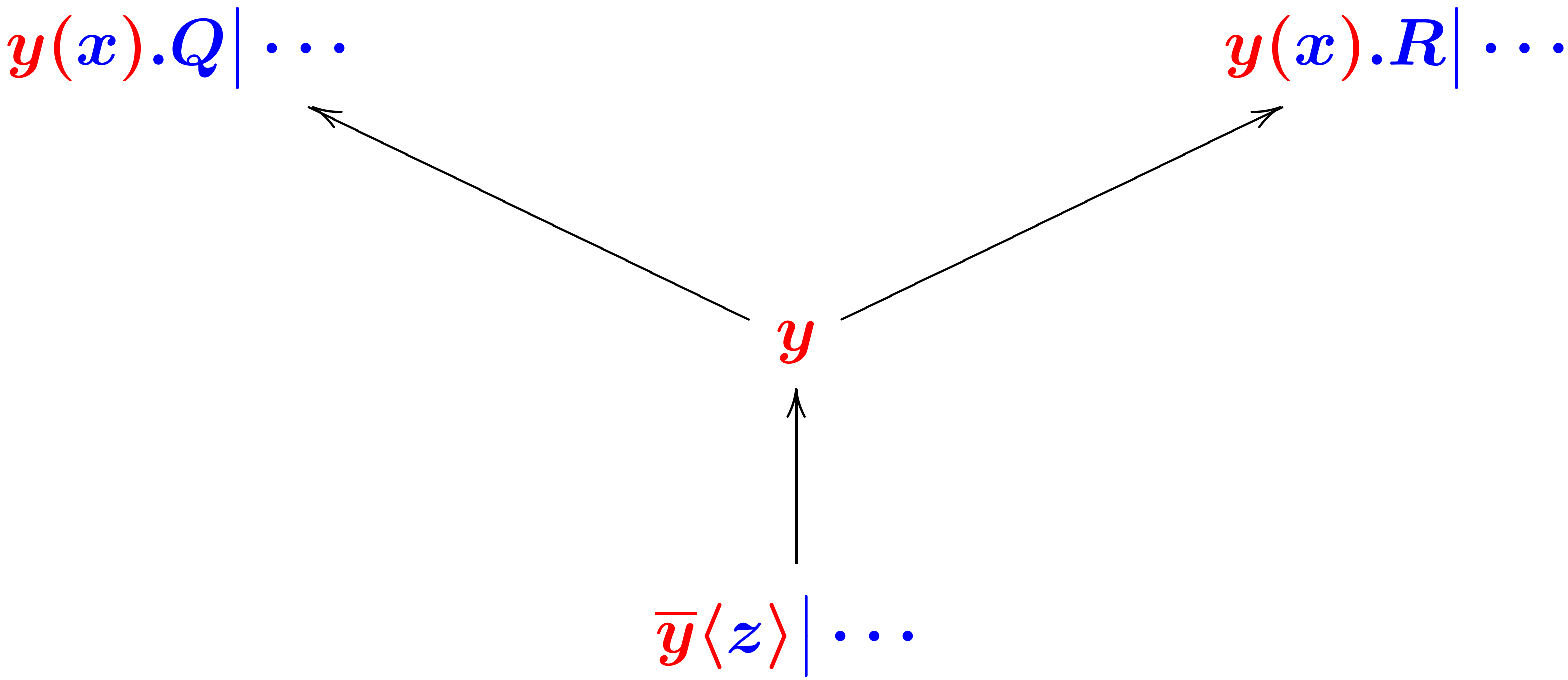
$$[\![x]\!](u) \stackrel{\text{def}}{=} \overline{x}\langle u\rangle$$

$$[\![\lambda x\, M]\!](u) \stackrel{\text{def}}{=} u(x,v).[\![M]\!]\langle v \rangle$$

$$[\![(M N)]\!](u) \stackrel{\text{def}}{=} (\nu v)\ \big(\ [\![M]\!]\langle v \rangle \mid (\nu x)\,\overline{v}\langle x,u\rangle \mid *x(u).[\![N]\!]\langle u \rangle\ \big)$$

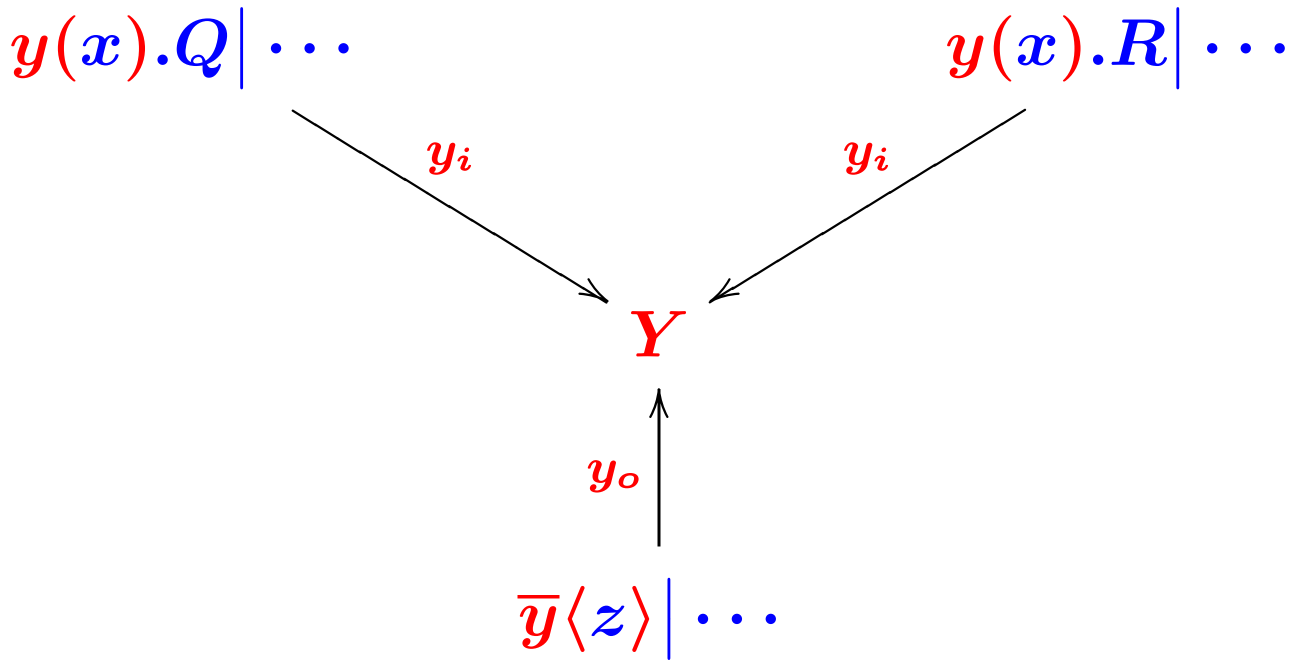
Try to evaluate/encode $(\cdots((M_0 N_1) N_2)\cdots)$

Distributed Implementation



- nearly every communication requires to solve a global consensus problem

Solution: Channel Managers



- **LOCALITY**: at most 1 receiver per channel
- $Y \stackrel{\text{def}}{=} *y_i(a).y_o(z).\overline{a}\langle z\rangle$

Locality syntactically: π^{def}

channel manager

residence site = creation site

avoids unnecessary global communications

$(\nu y)\, P$
$y(x).P$
$*P$

$$\textbf{def}\ \ y(x) = P\ \ \textbf{in}\ \ Q$$

$$(\nu y)\ (\ *\, y(x).P \ \mid\ Q\ )$$

channel managers are like function **def**initions
unique (replicated) receivers for the defined channels

Consequences (I)

. . . restricted π-notation . . .

REPLICATION

$$y(x).P \quad | \quad \overline{y}\langle z \rangle \qquad \textbf{non}$$

$$(\nu y)\,(\quad *y(x).P \quad | \quad \overline{y}\langle z \rangle \quad) \qquad \textbf{oui}$$

SHARING

$$*y(x).P_1 \quad | \quad *y(x).P_2 \quad | \quad \overline{y}\langle z \rangle \qquad \textbf{non}$$

$$(\nu y)\,(\quad *y(x).P \quad | \quad \overline{y}\langle z_1 \rangle \quad | \quad \overline{y}\langle z_2 \rangle \quad) \qquad \textbf{oui}$$

Consequences (II)

. . . restricted π-notation . . .

PREFIX-NESTING

$(\nu y)\ (\ \ *y(x).u(v).P\ \ )$ **non**

$(\nu y)\ (\ \ *y(x).\overline{u}\langle v\rangle\ \ )$ **oui**

$\longrightarrow$ in particular: INVERSION-OF-POLARITY

$(\nu y)\ (\ \ *y(x).x(u).P\ \ )$ **non**

$(\nu y)\ (\ \ *y(x).\overline{x}\langle u\rangle\ \ )$ **oui**

$(\nu y)\ (\ \ *y(x).\overline{u}\langle x\rangle\ \ )$ **oui**

Definition π^{def}

CORE SYNTAX with y, x channel (names):

D	$::=$	$y(x){=}P$					
P, Q	$::=$	$\mathbf{def}\ D\ \mathbf{in}\ Q$	$\mid$	$y(x)$	$\mid$	$P \vert Q$	

REDUCTION SEMANTICS

1 computational rule: $\mathbf{def}\ y(x){=}P\ \mathbf{in}\ (Q \mid y(z))$

$\longrightarrow \mathbf{def}\ y(x){=}P\ \mathbf{in}\ (Q \mid P[{}^{z}/_{x}])$

+ structural rules $\ldots$

Examples π^{def}

FORWARDER

$$\textbf{def } y(x) = u(x) \textbf{ in } y(z)$$

$$\longrightarrow \quad \textbf{def } y(x) = u(x) \textbf{ in } u(z)$$

APPLICATOR

$$\textbf{def } \mathrm{eval}(f, x) = f(x) \textbf{ in } \quad \mathrm{eval}(\mathrm{square}, 5) \quad | \cdots$$

$$\longrightarrow \quad \textbf{def } \mathrm{eval}(f, x) = f(x) \textbf{ in } \quad \mathrm{square}(5) \quad | \cdots$$

Expressiveness?

π^{def} is not expressive enough:

$$\textbf{def } D \textbf{ in } (P|Q) \triangleq \textbf{def } D \textbf{ in } P \mid \textbf{def } D \textbf{ in } Q$$

- no synchronization over parallel composition
- only local/functional computations
  by sending and receiving individual messages

Join-Synchronization: π_j

CORE SYNTAX with y, x, u, w channel (names):

$$D \quad ::= \quad \overbrace{y(x)|u(w)}^{J} = P$$

$$P, Q \quad ::= \quad \textbf{def } D \textbf{ in } Q \quad \Big| \quad y(x) \quad \Big| \quad P|Q$$

REDUCTION SEMANTICS generalization of π^{def}:

1 computational rule: $\textbf{def } J{=}P \textbf{ in } (Q|J\sigma)$

$\longrightarrow \textbf{def } J{=}P \textbf{ in } (Q|P\sigma)$

+ structural rules $\ldots$

Examples π_j (I)

Let D be defined as $y_1(x_1)|y_2(x_2) \; = \; P$.

$a)$ **def** D **in** $a(z_1)|b(z_2)|c(z_3)$

$\longrightarrow$

$b)$ **def** D **in** $y_1(z)$

$\longrightarrow$

$c)$ **def** D **in** $y_1(z_1)|y_2(z_2)|Q$

$\longrightarrow$

Examples π_j (II)

MULTIPLEXER

def $y(x)|u(w) = z(u, w)$ **in** $\ldots$

APPLICATOR

def apply$(f)|$args$(\tilde{w}) = f(\tilde{w})$ **in** $\ldots$

PRINTER-SPOOLER

def ready(printer) | job(doc) = printer(doc)

in ready(laser) | job(ps) | job(pdf)

$\longrightarrow \ldots$

Expressiveness!

$$[\![\textbf{def}\ y(x)|u(w)=P\ \textbf{in}\ Q]\!] \stackrel{\text{def}}{=} (\nu y, u)\ \big(y(x).u(w).[\![P]\!] \;\big|\; [\![Q]\!]\big)$$

$$[\![x(u)]\!] \stackrel{\text{def}}{=} \overline{x}\langle u\rangle$$

$$[\![P \mid Q]\!] \stackrel{\text{def}}{=} [\![P]\!] \mid [\![Q]\!]$$

$$[\![(\nu y)\ P]\!] \stackrel{\text{def}}{=} \textbf{def}\ y_o(x_o, x_i)|y_i(\kappa)=\kappa(x_o, x_i)\ \textbf{in}\ [\![P]\!]$$

$$[\![\overline{y}\langle z\rangle]\!] \stackrel{\text{def}}{=} y_o(z_o, z_i)$$

$$[\![y(x).P]\!] \stackrel{\text{def}}{=} \textbf{def}\ \kappa(x_o, x_i)=[\![P]\!]\ \textbf{in}\ y_i(\kappa)$$

π_j-calculus “ $\approx$ ” π_a-calculus