Concurrency Semantics
Week 9

Course Notes 2005
EPFL – I&C

Uwe Nestmann
Johannes Borgström

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7.13 Notation (Late transitions) We use the following sets of entities with corresponding meta-variables:

$$
\begin{array}{lllll}
\mathcal{N} & \textit{names} & a, b, c \ldots, x, y, z \\
\mathcal{A} & \textit{actions} & \pi ::= \tau \mid x(\vec{y}) \mid \overline{x}\langle\vec{y}\rangle \\
\mathcal{L} & \textit{labels} & \pi ::= \tau \mid x(\vec{y}) \mid (\boldsymbol{\nu}\vec{z})\,\overline{x}\langle\vec{y}\rangle
\end{array}
$$

where in *bound output* $(\boldsymbol{\nu}\vec{z})\,\overline{x}\langle\vec{y}\rangle$, we require $\tilde{z} \subseteq \tilde{y}$. We write $\overline{x}\langle\vec{y}\rangle$ for $(\boldsymbol{\nu}\vec{z})\,\overline{x}\langle\vec{y}\rangle$ when $\vec{z}$ is empty.

7.14 Definition (Late Operational Semantics) The LTS $(\mathcal{P}^\pi, \mathcal{T}^{\mathrm{L}})$ of sequential process expressions over $\mathcal{A}$ has $\mathcal{P}^\pi$ as states, and its transitions $\mathcal{T}^{\mathrm{L}}$ are precisely generated by the following rules:

$$(\text{PRE})\ \mu.P \xrightarrow{\mu} P$$

$$(\text{RES})\ \frac{P \xrightarrow{\mu} P'}{(\boldsymbol{\nu}c)\,P \xrightarrow{\mu} (\boldsymbol{\nu}c)\,P'} \text{ if } c \notin \mathrm{n}(\mu)$$

$$(\text{OPEN})\ \frac{P \xrightarrow{(\boldsymbol{\nu}\vec{b})\,\overline{a}\langle\vec{z}\rangle} P'}{(\boldsymbol{\nu}c)\,P \xrightarrow{(\boldsymbol{\nu}c\vec{b})\,\overline{a}\langle\vec{z}\rangle} P'} \text{ if } \vec{z} \ni c \notin \{a, \vec{b}\}$$

$$(\text{PAR}_1)\ \frac{P_1 \xrightarrow{\mu} P_1'}{P_1 \mid P_2 \xrightarrow{\mu} P_1' \mid P_2} \text{ if } \mathrm{bn}(\mu) \cap \mathrm{fn}(P_2) = \emptyset$$

$$(\text{CLOSE}_1)\ \frac{P_1 \xrightarrow{a(\vec{x})} P_1' \qquad P_2 \xrightarrow{(\boldsymbol{\nu}\vec{c})\,\overline{a}\langle\vec{b}\rangle} P_2'}{P_1 \mid P_2 \xrightarrow{\tau} (\boldsymbol{\nu}\vec{c})\,(\{^{\vec{b}}/_{\vec{x}}\}P_1' \mid P_2')} \text{ if } \Phi$$

$$\text{where } \Phi = \left(\ \{\vec{c}\} \cap \mathrm{fn}(P_1) = \emptyset \ \wedge\ |\vec{x}| = |\vec{b}|\ \right)$$

$$(\text{SUM}_1)\ \frac{P_1 \xrightarrow{\mu} P_1'}{P_1 + P_2 \xrightarrow{\mu} P_1'}$$

$$(\text{REP})\ \frac{P \mid \mathbf{!}\,P \xrightarrow{\mu} P'}{\mathbf{!}\,P \xrightarrow{\mu} P'} \qquad (\text{ALP})\ \frac{Q \xrightarrow{\mu} Q'}{P \xrightarrow{\mu} Q'} \text{ if } P =_\alpha Q$$

where the obvious symmetric counterparts for the rules (PAR_1), (CLOSE_1) and (SUM_1) are omitted.

7.15 Notation If R is a binary relation, then $R^=$ denotes its symmetric closure $R \cup R^{-1}$.

7.16 Definition (Distinction)

1. A *distinction* $D \subseteq (\mathcal{N} \times \mathcal{N})$ is a finite symmetric irreflexive relation on names.

2. Let $N \subseteq \mathcal{N}$. Then

$$N^{\neq} \stackrel{\text{def}}{=} \{ (x, y) \mid x, y \in N \wedge x \neq y \}.$$

3. A substitution σ *respects* a distinction D if $(x, y) \in D$ implies $\sigma x \neq \sigma y$.

4. A *D-congruence* is an equivalence that is preserved by those contexts that do not use the names in D as "hole-binding" names.

7.17 Definition (Strong Open Bisimulation)
The set $\{\sim^D \mid D \text{ is a distinction}\}$ is the largest family of symmetric relations such that if $P \sim^D Q$ and σ respects D, then

- if $\sigma P \xrightarrow{\mu} P'$ and μ is not a bound output, then there is Q' such that $\sigma Q \xrightarrow{\mu} Q'$ with $P' \sim^{\sigma D} Q'$.

- if $\sigma P \xrightarrow{(\boldsymbol{\nu}\vec{w})\,\overline{y}\langle\vec{z}\rangle} P'$, then there are $\vec{w}', Q'$ with $|\vec{w}'| = |\vec{w}|$ such that $\sigma P \xrightarrow{(\boldsymbol{\nu}\vec{w}')\,\overline{y}\langle\{\vec{w}'/\vec{w}\}\vec{z}\rangle} \{\vec{w}'/\vec{w}\}P'$ and $\sigma Q \xrightarrow{(\boldsymbol{\nu}\vec{w}')\,\overline{y}\langle\vec{z}\rangle} Q'$ with $\{\vec{w}'/\vec{w}\}P' \sim^{D'} Q'$ where

$$D' \stackrel{\text{def}}{=} \sigma D \cup \{\vec{w}'\}^{\neq} \cup (\{\vec{w}'\} \times \text{fn}(\sigma P, \sigma Q))^{\neq}$$

The weak version ($\approx^D$) is defined as usual by allowing for a weak simulating transition in each case.

7.18 Theorem $\sim^D$ and $\approx^D$ are *D-congruences*.