Concurrency Semantics
Week 6

Course Notes 2005
EPFL – I&C

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5 Observation Equivalence

5.1 Definition (Weak Transitions)
Given any LTS $(\mathcal{Q}, \mathcal{T})$.
Then, the *weak* transition relations $\Rightarrow$
and $\overset{\mu}{\Longrightarrow}$ (for $\mu \in \mathcal{A}$) are defined by:

1. $\Rightarrow \overset{\text{def}}{=} \overset{\tau}{\longrightarrow}{}^*$
2. $\overset{\mu}{\Longrightarrow} \overset{\text{def}}{=} \Rightarrow \overset{\mu}{\longrightarrow} \Rightarrow$

5.2 Definition (Weak Simulation)
Given any LTS $(\mathcal{Q}, \mathcal{T})$.
Let S be a binary relation over $\mathcal{Q}$.
Then S is said to be a *weak simulation*
if, whenever $P\ S\ Q$,

- if $P \overset{\tau}{\longrightarrow} P'$ then there is $Q' \in \mathcal{P}$
  such that $Q \Rightarrow Q'$ and $P'\ S\ Q'$.
- if $P \overset{\lambda}{\longrightarrow} P'$ then there is $Q' \in \mathcal{P}$
  such that $Q \overset{\lambda}{\Longrightarrow} Q'$ and $P'\ S\ Q'$.

Q weakly simulates P,
if there is a weak simulation S such that $P\ S\ Q$.

5.3 Lemma
Every strong simulation is also a weak simulation.

5.4 Definition (Weak Bisimulation)
A binary relation B is *a weak bisimulation*
if both B and its converse B^{-1} are weak simulations.
P and Q are *weakly bisimilar*, *weakly equivalent*,
or *observation equivalent*, written $P \approx Q$,
if there exists a weak bisimulation B with $P\ B\ Q$.

5.5 Remark $\approx = \bigcup\{\ B \mid B$ is weak bisimulation $\}$.

5.6 Proposition $P \sim Q$ implies $P \approx Q$.

5.7 Proposition (Weak Equivalence)

1. $\approx$ is itself a weak bisimulation.
2. $\approx$ is an equivalence relation.

5.8 Theorem
Weak equivalence $\approx$ is a process congruence.

5.9 Definition (Weak simulation up to $\sim$)
S is a *weak simulation up to* $\sim$
if, whenever $P \; S \; Q$,

- if $P \rightarrow P'$ then there is $Q' \in \mathcal{P}$
  such that $Q \Rightarrow Q'$ and $P' \sim S \sim Q'$.
- if $P \xrightarrow{\lambda} P'$ then there is $Q' \in \mathcal{P}$
  such that $Q \stackrel{\lambda}{\Longrightarrow} Q'$ and $P' \sim S \sim Q'$.

S is a *weak bisimulation up to* $\sim$
if its converse also has this property.

5.10 Proposition
If B is a (weak) bisimulation up to $\sim$ and $P \; B \; Q$,
then $P \approx Q$.

5.11 Theorem (Unique Solution of Equations)
Let $\vec{X} = X_1, X_2, \ldots$ be a (possibly infinite) sequence
of process variables. In the equations

$$\begin{array}{lll} X_1 & \approx & \mu_{11}.X_{k(11)} + \cdots + \mu_{1n_1}.X_{k(1n_1)} \\ X_2 & \approx & \mu_{21}.X_{k(21)} + \cdots + \mu_{2n_1}.X_{k(2n_1)} \\ \cdots & \approx & \cdots \end{array}$$

assume that $\mu_{ij} \neq \tau$. Then, up to $\approx$, there is a unique
sequence $P_1, P_2, \ldots$ of processes which satisfies the
equations.