Concurrency Semantics

Exercises 3

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1. Warmup

Let

$$P \stackrel{\text{def}}{=} (\boldsymbol{\nu} a)\ \big((\boldsymbol{\nu} b)\,(\overline{b}.b + \overline{a}.a \mid a.b) \bigm| (\boldsymbol{\nu} c)\,(a.\overline{b} + b.\overline{a})\big)$$

$$Q \stackrel{\text{def}}{=} (\boldsymbol{\nu} ac)\ \big((\overline{a}.a + \overline{c}.c \mid a.\overline{b} + b.\overline{a}) \bigm| a.c\big)$$

$$R \stackrel{\text{def}}{=} (\boldsymbol{\nu} ac)\ \big((a \mid \overline{b}) \bigm| a.c\big)$$

1. Prove (or at least give each structural congruence rule used) that P and Q are structurally congruent.

2. Prove that $P \ \rightarrow R$.

3. Using Propositions 3.11-13, give a process expression P', in sequential form (cf. Definition 2.2) and not containing any process identifiers, such that $P \sim P'$.

4. Given any process P, how many processes Q_i in standard form are there such that $P \equiv Q_i$? Motivate why.

2. Linked Buffers

Recall the boolean one- and two-place buffers.

$$
\begin{aligned}
\mathsf{Buf}^{(1)}(\, i_0, i_1, o_0, o_1 \,) &\stackrel{\text{def}}{=} i_0.\mathsf{Buf}^{(1)}_{\mathsf{0}}\langle\, i_0, i_1, o_0, o_1 \,\rangle + i_1.\mathsf{Buf}^{(1)}_{\mathsf{1}}\langle\, i_0, i_1, o_0, o_1 \,\rangle \\
\mathsf{Buf}^{(1)}_{\mathsf{i}}(\, i_0, i_1, o_0, o_1 \,) &\stackrel{\text{def}}{=} \overline{o_i}.\mathsf{Buf}^{(1)}\langle\, i_0, i_1, o_0, o_1 \,\rangle
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{Buf}^{(2)}(\, i_0, i_1, o_0, o_1 \,) &\stackrel{\text{def}}{=} i_0.\mathsf{Buf}^{(2)}_{\mathsf{0}}\langle\, i_0, i_1, o_0, o_1 \,\rangle + i_1.\mathsf{Buf}^{(2)}_{\mathsf{1}}\langle\, i_0, i_1, o_0, o_1 \,\rangle \\
\mathsf{Buf}^{(2)}_{\mathsf{i}}(\, i_0, i_1, o_0, o_1 \,) &\stackrel{\text{def}}{=} \overline{o_i}.\mathsf{Buf}^{(2)}\langle\, i_0, i_1, o_0, o_1 \,\rangle \\
&\quad + i_0.\mathsf{Buf}^{(2)}_{\mathsf{i0}}\langle\, i_0, i_1, o_0, o_1 \,\rangle + i_1.\mathsf{Buf}^{(2)}_{\mathsf{i1}}\langle\, i_0, i_1, o_0, o_1 \,\rangle \\
\mathsf{Buf}^{(2)}_{\mathsf{ij}}(\, i_0, i_1, o_0, o_1 \,) &\stackrel{\text{def}}{=} \overline{o_i}.\mathsf{Buf}^{(2)}_{\mathsf{j}}\langle\, i_0, i_1, o_0, o_1 \,\rangle
\end{aligned}
$$

Last week you were asked to construct a two-place buffer from two one-place buffers. A simple way of doing this is by linking two one-place buffers together, so that the output of one becomes the input of the other.

A general linking operation can be defined in this way:

$$P \;{}^{\tilde{x}}\!\frown^{\tilde{y}}\; Q \stackrel{\text{def}}{=} (\boldsymbol{\nu}\tilde{z})\,(P\{^{\tilde{z}}/_{\tilde{x}}\} \big| Q\{^{\tilde{z}}/_{\tilde{y}}\}),$$

for some $\tilde{z}$ where $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ have the same length, and all names in $\tilde{z}$ are fresh with respect to both P and Q. As an example we show how two one-place buffers can be composed into a two-place buffer.

$$
\begin{aligned}
\mathsf{Buf}^{(1)}\langle\, i_0, i_1, o_0, o_1 \,\rangle \;{}^{o_0,o_1}\!\frown^{i_0,i_1}\; \mathsf{Buf}^{(1)}\langle\, i_0, i_1, o_0, o_1 \,\rangle = \\
(\boldsymbol{\nu} c_0, c_1)\,(\mathsf{Buf}^{(1)}\langle\, i_0, i_1, c_0, c_1 \,\rangle \big| \mathsf{Buf}^{(1)}\langle\, c_0, c_1, o_0, o_1 \,\rangle)
\end{aligned}
$$

1. Is this composed two-place buffer bisimilar to $\mathsf{Buf}^{(2)}\langle\, i_0, i_1, o_0, o_1 \,\rangle$? Prove that your response is correct.

2. If necessary, modify the defining equation of the $\mathsf{Buf}^{(2)}_{\mathsf{x}}(\, i_0, i_1, o_0, o_1 \,)$ to make the two processes bisimilar, and prove the bisimilarity.

3. Think about how to generalize this composition to n-place buffers.