Concurrency Semantics
Exercises 2

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1. Warmup

1. Write down a clash-free process that is α-equivalent to

$$(\boldsymbol{\nu} a)\,(a.(\boldsymbol{\nu} a)\,a.\mathbf{0} \mid (\boldsymbol{\nu} a)\,a.\mathbf{0}).$$

   Prove that the two processes are α-equivalent.

2. Write down the silent (τ) transitions of the process

$$(\boldsymbol{\nu} a)\,(a.Q_1 + b.Q_2 \mid \overline{a}.\mathbf{0}) \mid \overline{b}.R_1 + \overline{a}.R_2.$$

   Give a formal derivation for one of them.

3. Prove that any two α-equivalent processes are bisimilar.

2. Transitions

Prove the following propositions.

1. If $P \xrightarrow{\mu} P'$ then $\mathrm{fn}(P') \cup \mathrm{fn}(\mu) \subseteq \mathrm{fn}(P)$.
2. If $P \xrightarrow{\mu} P'$ and σ is a substitution (with finite support) then $\sigma P \xrightarrow{\sigma\mu} \sigma P'$.

3. Buffers

Recall the boolean single- and two-place buffers

$$\begin{aligned}
\mathsf{Buf}^{(1)}(i_0,i_1,o_0,o_1) &\stackrel{\text{def}}{=} i_0.\mathsf{Buf}_0^{(1)}(i_0,i_1,o_0,o_1) + i_1.\mathsf{Buf}_1^{(1)}(i_0,i_1,o_0,o_1)\\
\mathsf{Buf}_i^{(1)}(i_0,i_1,o_0,o_1) &\stackrel{\text{def}}{=} \overline{o_i}.\mathsf{Buf}^{(1)}(i_0,i_1,o_0,o_1)
\end{aligned}$$

$$\begin{aligned}
\mathsf{Buf}^{(2)}(i_0,i_1,o_0,o_1) &\stackrel{\text{def}}{=} i_0.\mathsf{Buf}_0^{(2)}(i_0,i_1,o_0,o_1) + i_1.\mathsf{Buf}_1^{(2)}(i_0,i_1,o_0,o_1)\\
\mathsf{Buf}_i^{(2)}(i_0,i_1,o_0,o_1) &\stackrel{\text{def}}{=} \overline{o_i}.\mathsf{Buf}^{(2)}(i_0,i_1,o_0,o_1)\\
&\quad + i_0.\mathsf{Buf}_{i0}^{(2)}(i_0,i_1,o_0,o_1) + i_1.\mathsf{Buf}_{i1}^{(2)}(i_0,i_1,o_0,o_1)\\
\mathsf{Buf}_{ij}^{(2)}(i_0,i_1,o_0,o_1) &\stackrel{\text{def}}{=} \overline{o_i}.\mathsf{Buf}_j^{(2)}(i_0,i_1,o_0,o_1)
\end{aligned}$$

1. Construct a two-place buffer from two one-place buffers.
2. Is your two-place buffer bisimilar to the one defined above?
   Prove that your response is correct.
3. If necessary, modify the definition above of the two-place buffer to make the two processes bisimilar, and prove the bisimilarity.