Concurrency: Theory, Languages and Programming

– Pi Calculus –

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Redexes

There are (at least) two reasons for studying LTSs (as opposed to mere reductions as in the -calculus):

the emphasis in on *interaction* with other programs

redexes in a concurrent program are usually distributed over terms, not juxtaposed as in -calculus.

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redexes are the “source” of reductions or internal transitions, visible as the pattern in the conclusion of either the -rule (in) or the COMM-rule (in [VP]CCS).

Unbounded Structures: Stacks (I)

"specification":
stored values are encoded in the index of the identifier

needs an unbounded number of process identifiers . . .

does not exploit "concurrency inside"

empty push pop

Stack

Stack push Stack $\overline{\text{empty}}$ Stack

Stack push Stack $\overline{\text{pop}}$ Stack

Unbounded Structures: Stacks (II)

"implementation":
using a **chain** of individual cells for the stored values

cells can have one of the following states:

: nothing is left in the stack accessible through this cell

: a cell containing value

: nothing left in this particular cell,
but maybe beyond on "the right of it" . . .

Unbounded Structures: Stacks (III)

push	empty	pop	not	drop	pull

	not drop pull	push empty pop
push	$\overline{\text{empty}}$	
push	$\overline{\text{pop}}$	
pull	drop	

Calculate the states for the transition sequence

push push pop

and "stabilize" the remainder.

Compare Stack and . . .

Turing Power

A **Turing-machine** consists of:

- a finite alphabet of symbols
- an infinite tape
- a finite control mechanism
- movement or r/w-head to left or right

A Turing-machine can be nicely simulated with concurrent processes by two stacks (the tape). Neither an infinite alphabet nor infinite summation is necessary for this. [Milner 89]

1. The calculus of concurrent process expressions is *Turing-powerful*.
2. The halting problem for some “Turing machine” TM can be encoded as the *existence* of an infinite sequence TM .

Unbounded Structures: Stacks (IV)

Some criticism:

’s cannot be reused for storing *new* values (neither inner nor outer ’s!).

’s are never “used”, pile up and stay around. (Note that, although “=” , explicit garbage collection would be required.)

Unbounded Structures: Stacks (V)

push	$\overline{\text{empty}}$		
push		$\overline{\text{pop}}$	$\overline{\text{not}}$
pull	drop	push	

What are the problems of this “implementation”?

Expressiveness

Although Turing-powerful, concurrent process expressions are, in some particular sense, not expressive enough:
it is not possible to *cut out unusable (=dead) cells* .

If we had the possibility to *dynamically change the interconnection structure* among process components, then cells could *drop out* by connecting their left and right neighbors together.

One way to do this is the transmission of "*channels over channels*".

Name-Passing Syntax

negative actions $\bar{\ }$: *send name* *over name* .

positive actions : *receive any name, say* , *over name* *and “bind the result” to name* .

Binding results in *substitution*
of the formal parameter by the actual parameter .

polyadic communication $\bar{\ }$ and (pairwise different)
transmit many values at a time.

Syntax Conventions

$\mathcal{N}$ names

$\mathcal{A}$ actions

finite sequences $\ldots$

All values/variables/channels are just names.

Parentheses usually indicate bindings.
Angled brackets are often omitted.

parametric processes with defining equations
are modeled via the more primitive notion
of replication and name-passing

Pi Calculus

Definition: The set of -calculus proc. exp. is defined (precisely) by the following syntax:

M $\square$

M $\square$ M M

We use to stand for process expressions.

abbreviates

$\sum$ abbreviates

Mobility ? “Flowgraphs” !

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Assume that .

Depict the transition

as a flow graph (with scopes) and verify it using the reaction and congruence rules.

Example: Hand-Over Protocol

$$\begin{array}{lllllll}
\mathrm{Car} & \mathrm{talk} & \mathrm{swch} & \overline{\mathrm{talk}} & \mathrm{Car} & \mathrm{talk} & \mathrm{swch} \\
\mathrm{Base} & & & \mathrm{talk} & \mathrm{Base} & \mathrm{give} & \overline{\mathrm{swch}} \quad \mathrm{Idle} \\
\mathrm{Idle} & & & \mathrm{alrt} & \mathrm{Base} & & \\
\mathrm{Ctre} & & & \overline{\mathrm{give}} & \mathrm{talk} & \mathrm{swch} & \overline{\mathrm{alrt}} \quad \mathrm{Ctre} \\
\mathrm{Syst} & & & \multicolumn{4}{l}{\left(\mathrm{Car} \quad \mathrm{talk} \quad \mathrm{swch} \quad \mathrm{Base} \quad \mathrm{Idle} \quad \mathrm{Ctre} \right)}
\end{array}$$

Exercise: Observe that $\mathrm{Syst} \qquad \mathrm{Syst}$

Exercise: Overtaking Cars

A car on a road is connected to its back and front neighbor through and , respectively, while just represents its identifier.

The road is assumed to be infinite, so we ignore any boundary problem, and it is static in the sense that no cars may enter or leave the road.

Define such that a car may overtake another car. Beware of deadlocks and nested overtake attempts. You are not allowed to change the parameter of instances of .

Car

Fast

Slow

LTS: Prefixes

actions –

(TAU) (OUT) –

–

(INP) $\mathcal{N}$

–

(COMM)

LTS: Restriction

(RES)

(OPEN)

The label on transition is called *bound output*.
(Invariant: .)

LTS: Parallel Composition

$$\text{(PAR)}\ \frac{}{}$$

$$\text{(CLOSE)}\ \frac{}{}$$

LTS: Miscellaneous

(SUM) ——————————

(REP) ——————————

(ALP) ————————

Recursion

, where

can be used in:

can be modeled through:

1. invent to stand for
2. for any ,
   let denote the result of replacing any call by ¯
3. replace by

Booleans

True –

False 

If foo bar –  

Check that for all :

True If foo bar “=” 

False If foo bar “=” 

Encoding Tuples

Think about:

Encoding Synchrony

Encoding Summation

$\sum$

$\sum$

Encoding Lambda-Calculus

M M

M M

Try to evaluate/encode M

“Final Words”

name-passing vs. **value-pasing**

better pragmatics

natural programming idioms

semantic foundations for all (?) major programming styles

. . .

security protocols via (s)pi-calculus
(research at LAMP2 . . .)