Concurrency: Theory, Languages and Programming

– CCS Example –

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$\mathcal{N}$	channels	$a, b, c \ldots$
$\mathcal{V}$	values	v, w
$\mathcal{X}$	variables	x, y, z
$\mathcal{A}$	actions	$\mu \ ::= \ \overline{a}\langle v\rangle \ \mid \ a(x) \ \mid \ \tau$

"negative" actions $\overline{a}\langle v\rangle$: *send name* v *over channel* a.

"positive" actions $a(x)$: *receive any value, say* v*, over channel* a *and "bind the result" to variable* x.

Binding results in *substitution* $[{}^{v}/_{x}]$
of the formal parameter x by the actual parameter v.

polyadic communication $\overline{a}\langle \vec{v}\rangle$ and $a(\vec{x})$ (with $\vec{x}$ pairwise different) *transmit many values at a time.*

Value-Passing: Semantics I

directly: via LTS

$\ldots$ TAU: $\tau.P \xrightarrow{\tau} P$ OUT: $\overline{a}\langle v\rangle.P \xrightarrow{\overline{a}\langle v\rangle} P$

$$\text{INP: } \frac{v \in \mathcal{V}}{a(x).P \xrightarrow{av} [{}^{v}/_{x}]P}$$

$$\text{COMM: } \frac{P \xrightarrow{\overline{a}\langle v\rangle} P' \qquad Q \xrightarrow{av} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q'}$$

Value-Passing: Semantics II

indirectly: via translation

$$\begin{array}{lcl}
[\![\ ]\!] : \mathcal{P}^{\mathsf{VP}} & \to & \mathcal{P} \\
\hline
[\![\, \overline{a}\langle v\rangle.P \,]\!] & \stackrel{\text{def}}{=} & \overline{a_v}.[\![\, P \,]\!] \\
[\![\, a(x).P \,]\!] & \stackrel{\text{def}}{=} & \sum\limits_{v \in \mathcal{V}} a_v.[\![\, [{}^{v}/_{x}]P \,]\!] \\
\vdots & & \\
[\![\, P_1 \mid P_2 \,]\!] & \stackrel{\text{def}}{=} & [\![\, P_1 \,]\!] \mid [\![\, P_2 \,]\!] \\
\vdots & & \\
[\![\, A\langle \vec{v}\,\rangle \,]\!] & \stackrel{\text{def}}{=} & A\langle \vec{v}\,\rangle
\end{array}$$

$\mathcal{N}$	$:=$	$\{$ in, out $\}$
$\mathcal{V}$	$:=$	$\{ 0, 1 \}$
s	$\in$	$\{\epsilon\} \cup \mathcal{V}$
$\vec{a}$	$:=$	in, out
$\mathsf{Buff}_s^{(1)}$	:	1-place buffer containing s
$\mathsf{Buff}_\epsilon^{(1)}(\,\vec{a}\,)$	$\stackrel{\text{def}}{=}$	$\mathsf{in}(x).\mathsf{Buff}_x^{(1)}\langle\,\vec{a}\,\rangle$
$\mathsf{Buff}_v^{(1)}(\,\vec{a}\,)$	$\stackrel{\text{def}}{=}$	$\overline{\mathsf{out}}\langle v\rangle.\mathsf{Buff}_\epsilon^{(1)}\langle\,\vec{a}\,\rangle$

- Observe how much nicer name/value-passing is :-)

- $(\boldsymbol{\nu} x)\, P$ and $a(x).P$ **bind** x in P
- x occurs **bound** in P, if it occurs in a subterm $(\boldsymbol{\nu} x)\, Q$ or $a(x).P$ of P
- x occurs **free** in P, if it occurs without enclosing $(\boldsymbol{\nu} x)\, Q$ or $a(x).P$ in P
- Note the use of parentheses (round brackets).
- Define $\mathrm{fn}(P)$ and $\mathrm{bn}(P)$ inductively on $\mathcal{P}$ (sets of free/bound names of P) ...

Scheduler, Informally [Mil99, § 3.6]

- a set of processes $P_i, 1 \leq i \leq n$ is to be scheduled
- P_i starts by signalling $\overline{a_i}$ to the scheduler
- P_i completes by signalling $\overline{b_i}$ to the scheduler
- each P_i *must not* run two tasks at a time
- tasks of different P_i *may* run at the same time
- a_i are required to occur cyclically (initially, 1 starts)
- for each i, a_i and b_i must occur cyclically
- maximal "progress":
  the scheduling must permit any of the buttons to be pressed at any time provided (1) and (2) are not violated.

Formal "Implementation" [§ 7.3]

$A(\,a,b,c,d\,)$	$\stackrel{\text{def}}{=}$	$a.c.b.\overline{d}.A$
$A(\,a,b,c,d\,)$	$\stackrel{\text{def}}{=}$	$a.C\langle\,a,b,c,d\,\rangle$
$C(\,a,b,c,d\,)$	$\stackrel{\text{def}}{=}$	$c.B\langle\,a,b,c,d\,\rangle$
$B(\,a,b,c,d\,)$	$\stackrel{\text{def}}{=}$	$b.D\langle\,a,b,c,d\,\rangle$
$D(\,a,b,c,d\,)$	$\stackrel{\text{def}}{=}$	$\overline{d}.A\langle\,a,b,c,d\,\rangle$
$\vec{a} := a_1 \ldots, a_n,\ \vec{b} := b_1 \ldots, b_n\ \ \vec{c} := c_1 \ldots, c_n$		
$A_i(\,\vec{a},\vec{b},\vec{c}\,)$	$\stackrel{\text{def}}{=}$	$A\langle\,a_i,b_i,c_i,c_{i\ominus_n 1}\,\rangle$
$B_i(\,\vec{a},\vec{b},\vec{c}\,)$	$\stackrel{\text{def}}{=}$	$B\langle\,a_i,b_i,c_i,c_{i\ominus_n 1}\,\rangle$
$C_i(\,\vec{a},\vec{b},\vec{c}\,)$	$\stackrel{\text{def}}{=}$	$C\langle\,a_i,b_i,c_i,c_{i\ominus_n 1}\,\rangle$
$D_i(\,\vec{a},\vec{b},\vec{c}\,)$	$\stackrel{\text{def}}{=}$	$D\langle\,a_i,b_i,c_i,c_{i\ominus_n 1}\,\rangle$
$S(\,\vec{a},\vec{b}\,)$	$\stackrel{\text{def}}{=}$	$(\boldsymbol{\nu}\vec{c})\,\big(\,A_1\langle\,\vec{a},\vec{b},\vec{c}\,\rangle \mid D_2\langle\,\vec{a},\vec{b},\vec{c}\,\rangle \mid \cdots \mid D_n\langle\,\vec{a},\vec{b},\vec{c}\,\rangle\,\big)$