Concurrency: Theory, Languages and Programming

– CCS –

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Plan

- ☐ Session 4
  - from λ-calculus to CCS: towards concurrency
  - Structural Operational Semantics (SOS)
- ☐ Session 5
  - examples ...
  - ... using the Scala-library
- ☐ Session 6
  - from CCS to π-calculus: pragmatics, syntax, semantics
  - more SOS
- ☐ Session 7
  - examples ...
  - ... using the Scala-library

Foundational Calculi?

We are interested in the foundations of programming.
We use "foundational" mini-languages as vehicles that guide our intuition and style of expression. When does such a mini-language deserve to be called a "calculus"?

- few primitives
- mathematically tractable
  - *calculate* computational steps
  - notion of equivalence
- *computationally complete* (Turing, URM, GOTO, . . .)
- "*naturally complete*": design of programming languages
  - easily "extensible" via *encodings*
  - *higher-order* principles

Concurrency?

- parallelism: ≥ 1 independent threads of control
- distribution:
  - logical concurrency
  - physical concurrency
  - failures
- synchronization / cooperation / coordination
  communication

$\Longrightarrow$ foundational calculus for (just) concurrency ?

λ-Calculus

- **Syntax** (for example)
  a BNF-grammar generates the set of expressions ...

$$M, N \quad ::= \quad x \quad \Big| \quad \lambda x.N \quad \Big| \quad MN$$

- **Semantics** (for example)
  a set of inference rules generates (and controls) the possible reductions of terms

$$(\beta)\ \frac{}{(\lambda x.N)M \ \rightarrow [M/x]N}$$

$$\text{(FUN)}\ \frac{M \ \rightarrow M'}{MN \ \rightarrow M'N} \qquad \text{(ARG)}\ \frac{N \ \rightarrow N'}{MN \ \rightarrow MN'}$$

Functional vs Concurrent

- functional / reduction systems:
  - reduce a term to value form
  - *only* the resulting value is interesting
  - observation after termination
- concurrent / reactive systems:
  - describe the possible interactions *during* evaluation
  - the resulting value is *not* (necessarily) interesting
  - observation through and during interaction

The notion of *interaction* (*communication*) is important !

Hoare (CSP) and Milner (CCS) proposed handshake-communication as the primitive form of interaction.

Functional vs Concurrent

	functional	concurrent
determinism	possible	?
confluence	wanted/needed	?
termination	?	?
foundation	λ	CCS, π, (Petri nets, . . .)
ff-language	ML, Scala, . . .	Pict, Join, Scala, . . .

CCS

$\mathcal{I}$	process identifiers	$A, B \ldots$
$\mathcal{N}$	names	$a, b, c \ldots$
$\overline{\mathcal{N}}$	co-names	$\overline{a}, \overline{b}, \overline{c} \ldots$
$\mathcal{L}$	labels (buttons)	metavariables $\lambda \ldots \in \mathcal{L} := \mathcal{N} \cup \overline{\mathcal{N}}$
$\mathcal{A}$	actions	metavariables $\mu, \beta \ldots \in \mathcal{L} \cup \{\tau\}$

- visible/external actions: labels
- invisible/internal actions: τ
- **finite sequences** $\vec{a}$ for *names* $a_1 \ldots, a_n$ (*not co*-names!)
- **parametric processes** $A\langle\, a, c \,\rangle$ with *name* parameters (neither *co*-names, nor labels, . . .)

Definition: The sets $\mathcal{P}^{\mathrm{seq}}$ and $\mathcal{M}^{\mathrm{seq}}$ of sequential process expressions is defined (precisely) by the following BNF-syntax:

$$\begin{array}{lcl} P & ::= & A\langle\,\vec{a}\,\rangle \;\mid\; M \\ M & ::= & \mathbf{0} \;\mid\; \mu.P \;\mid\; M + M \end{array}$$

We use $P, Q, P_i \ldots$ to stand for *process expressions*, while M, M_i always stand for *choices* or *summations*. We also use the abbreviation

$$\sum_{i \in I} \mu_i.P_i := \mu_1.P_1 + \ldots + \mu_n.P_n$$

where I is the finite indexing set $\{1 \ldots, n\}$. Note that then the order of summands is not fixed.

Sequential Process Expressions (II)

- each process identifier A is assumed to have a **defining equation** (note the brackets)

  $$A(\,\vec{a}\,) \stackrel{\mathrm{def}}{=} M$$

  where M is a summation, $\vec{a}$ is (or: includes) $\mathrm{fn}(M)$. Note: $\vec{a}$ does only include *names*!
- $\mathrm{fn}(P)$: the set of all of the **(free) names** of P
- $A\langle\,\vec{b}\,\rangle$ means the same as $[\vec{b}/\vec{a}]M$
- **substitution** $[\vec{b}/\vec{a}]P$ (for matching $\vec{b}$ and $\vec{a}$) replaces *all* occurrences of a_i in P by b_i.

Definition: The set $\mathrm{fn}(P)$ is defined inductively by:

$$\mathrm{fn}(\mu) \stackrel{\text{def}}{=} \begin{cases} \{b\} & \text{if } \mu = b \\ \{b\} & \text{if } \mu = \overline{b} \\ \emptyset & \text{if } \mu = \tau \end{cases}$$

$$\mathrm{fn}(\mathbf{0}) \stackrel{\text{def}}{=} \emptyset$$

$$\mathrm{fn}(\mu.P) \stackrel{\text{def}}{=} \mathrm{fn}(\mu) \cup \mathrm{fn}(P)$$

$$\mathrm{fn}(M_1 + M_2) \stackrel{\text{def}}{=} \mathrm{fn}(M_1) \cup \mathrm{fn}(M_2)$$

$$\mathrm{fn}(A\langle \vec{a} \rangle) \stackrel{\text{def}}{=} \{\vec{a}\}$$

Substitution, Inductively

Definition:

$$[b/c]\mu \stackrel{\text{def}}{=} \begin{cases} b & \text{if } \mu = c \\ \overline{b} & \text{if } \mu = \overline{c} \\ \mu & \text{otherwise} \end{cases}$$

$$[b/c]\mathbf{0} \stackrel{\text{def}}{=} \mathbf{0}$$

$$[b/c](\mu.P) \stackrel{\text{def}}{=} [b/c]\mu.[b/c]P$$

$$[b/c](M_1 + M_2) \stackrel{\text{def}}{=} [b/c]M_1 + [b/c]M_2$$

$$[b/c](A\langle \vec{a} \rangle) \stackrel{\text{def}}{=} A\langle [b/c]\vec{a} \rangle$$

Definition:

Let $\vec{b} = b_1 \ldots, b_n$ and $\vec{c} = c_1 \ldots, c_n$.

$$[\vec{b}/\vec{c}]\mu \stackrel{\text{def}}{=} \begin{cases} b_i & \text{if } \exists 1 \leq i \leq n \text{ with } \mu = c_i \\ \overline{b_i} & \text{if } \exists 1 \leq i \leq n \text{ with } \mu = \overline{c_i} \\ \ldots & \text{otherwise} \end{cases}$$

$$[\vec{b}/\vec{c}]\mathbf{0} \stackrel{\text{def}}{=} \mathbf{0}$$

$$[\vec{b}/\vec{c}](\mu.P) \stackrel{\text{def}}{=} [\vec{b}/\vec{c}]\mu.[\vec{b}/\vec{c}]P$$

$$[\vec{b}/\vec{c}](M_1 + M_2) \stackrel{\text{def}}{=} [\vec{b}/\vec{c}]M_1 + [\vec{b}/\vec{c}]M_2$$

$$[\vec{b}/\vec{c}](A\langle\, \vec{a}\,\rangle) \stackrel{\text{def}}{=} A\langle\, [\vec{b}/\vec{c}]\vec{a}\,\rangle$$

Example: 1-Place Binary Buffer

$$\begin{array}{rcl} \mathcal{N} & := & \{\, \mathsf{in}_i, \mathsf{out}_i \mid i \in \{0,1\} \,\} \\ s & \in & \{\epsilon, 0, 1\} \\ \vec{a} & := & \mathsf{in}_0, \mathsf{in}_1, \mathsf{out}_0, \mathsf{out}_1 \\ \mathsf{Buff}_s^{(1)} & : & \text{1-place buffer containing } s \end{array}$$

$$\mathsf{Buff}^{(1)}(\,\vec{a}\,) \stackrel{\text{def}}{=} \textstyle\sum_{i\in\{0,1\}} \mathsf{in}_i.\mathsf{Buff}_i^{(1)}\langle\, \vec{a}\,\rangle$$

$$\mathsf{Buff}_i^{(1)}(\,\vec{a}\,) \stackrel{\text{def}}{=} \overline{\mathsf{out}_i}.\mathsf{Buff}^{(1)}\langle\, \vec{a}\,\rangle$$

$\mathcal{N}$	$:=$	$\{\ \mathsf{in}_i, \mathsf{out}_i \mid i \in \{0,1\}\ \}$
s	$\in$	$\{\epsilon, 0, 1, 00, 01, 10, 11\}$
$\vec{a}$	$:=$	$\mathsf{in}_0, \mathsf{in}_1, \mathsf{out}_0, \mathsf{out}_1$
$\mathsf{Buff}_s^{(2)}$	:	2-place buffer containing s
$\mathsf{Buff}^{(2)}(\ \vec{a}\ )$	$\stackrel{\text{def}}{=}$	$\sum_{i\in\{0,1\}} \mathsf{in}_i.\mathsf{Buff}_i^{(2)}\langle\ \vec{a}\ \rangle$
$\mathsf{Buff}_i^{(2)}(\ \vec{a}\ )$	$\stackrel{\text{def}}{=}$	$\overline{\mathsf{out}_i}.\mathsf{Buff}^{(2)}\langle\ \vec{a}\ \rangle + \sum_{j\in\{0,1\}} \mathsf{in}_j.\mathsf{Buff}_{ji}^{(2)}\langle\ \vec{a}\ \rangle$
$\mathsf{Buff}_{ij}^{(2)}(\ \vec{a}\ )$	$\stackrel{\text{def}}{=}$	$\overline{\mathsf{out}_j}.\mathsf{Buff}_i^{(2)}\langle\ \vec{a}\ \rangle$

- modify $\mathsf{Buff}_s^{(2)}$ to release values in either order
- write an analogous definition for $\mathsf{Buff}_s^{(3)}$ ...

Labeled Transition Systems

Definition:
An **LTS** $(\mathcal{Q}, \mathcal{T})$ over an **action alphabet** $\mathcal{A}$:

- a set of **states** $\mathcal{Q} = \{q_0, q_1 \ldots\}$
- a ternary **transition relation** $\mathcal{T} \subseteq (\mathcal{Q} \times \mathcal{A} \times \mathcal{Q})$

A transition $(q, \mu, q') \in \mathcal{T}$ is also written $q \stackrel{\mu}{\longrightarrow} q'$.

If $q \stackrel{\mu_1}{\longrightarrow} q_1 \cdots \stackrel{\mu_n}{\longrightarrow} q_n$ we call q_n a **derivative** of q.

LTSs are automata, but ignoring starting and accepting states.
Transition Graphs are useful ...

Definition: The LTS $(\mathcal{P}^{\mathrm{seq}}, \mathcal{T})$ of sequential process expressions over $\mathcal{A}$ has $\mathcal{P}^{\mathrm{seq}}$ as states, and its transitions $\mathcal{T}$ are precisely generated by the following rules:

$$\textsc{pre}: \ \mu.P \xrightarrow{\mu} P$$

$$\textsc{sum}_1: \ \frac{M_1 \xrightarrow{\mu} M_1'}{M_1+M_2 \xrightarrow{\mu} M_1'} \qquad \textsc{sum}_2: \ \frac{M_2 \xrightarrow{\mu} M_2'}{M_1+M_2 \xrightarrow{\mu} M_2'}$$

$$\textsc{def}: \ \frac{[\vec{b}/\vec{a}]M_A \xrightarrow{\mu} P'}{A\langle \vec{b} \rangle \xrightarrow{\mu} P'} \ \textsc{if} \ A(\vec{a}) \stackrel{\mathrm{def}}{=} M_A$$

Note that transition under prefix is not allowed/included.

Concurrent Process Expressions (I)

Definition: The set $\mathcal{P}$ of concurrent process expressions is defined (precisely) by the following BNF-syntax:

$$\begin{array}{rcl} P & ::= & A\langle \vec{a} \rangle \ \mid \ M \ \mid \ P|P \ \mid \ (\boldsymbol{\nu} a)\, P \\ M & ::= & \mathbf{0} \ \mid \ \alpha.P \ \mid \ M+M \end{array}$$

We use P, Q, P_i to stand for process expressions.

- $(\boldsymbol{\nu} a)\, P$ restricts the scope of a to P
- $(\boldsymbol{\nu} ab)\, P$ abbreviates $(\boldsymbol{\nu} a)\, (\boldsymbol{\nu} b)\, P$

- precedence: unary binds tighter than binary

$$\begin{aligned}(\boldsymbol{\nu} a)\, P \mid Q &= ((\boldsymbol{\nu} a)\, P) \mid Q \\ a.P + M &= (a.P) + M\end{aligned}$$

$$[a/b]M_1 + M_2 = ([a/b]M_1) + M_2$$

- what about:

$$\begin{aligned}P \mid Q + R &\stackrel{?}{=} (P \mid Q) + R \\ P \mid Q + R &\stackrel{?}{=} P \mid (Q + R)\end{aligned}$$

Bound and Free Names

- $(\boldsymbol{\nu} a)\, P$ **binds** a in P
- a occurs **bound** in P,
  if it occurs in a subterm $(\boldsymbol{\nu} a)\, Q$ of P
- a occurs **free** in P,
  if it occurs without enclosing $(\boldsymbol{\nu} a)\, Q$ in P
- Define $\mathrm{fn}(P)$ and $\mathrm{bn}(P)$ inductively on $\mathcal{P}$
  (sets of free/bound names of P):

$$\begin{aligned}\mathrm{fn}(P_1|P_2) &\stackrel{\text{def}}{=} \mathrm{fn}(P_1) \cup \mathrm{fn}(P_2) \\ \mathrm{bn}(P_1|P_2) &\stackrel{\text{def}}{=} \mathrm{bn}(P_1) \cup \mathrm{bn}(P_2) \\ &\cdots \\ \mathrm{fn}((\boldsymbol{\nu} a)\, P) &\stackrel{\text{def}}{=} \mathrm{fn}(P) \setminus \{a\} \\ \mathrm{bn}((\boldsymbol{\nu} a)\, P) &\stackrel{\text{def}}{=} \mathrm{bn}(P) \cup \{a\}\end{aligned}$$

α-Conversion & Substitution

- **substitution** $[\vec{b}/\vec{a}]P$ (for matching $\vec{b}$ and $\vec{a}$)
  replaces *all* **free** occurrences of a_i in P by b_i.

  $[b/a](\boldsymbol{\nu} b)\, b.a = ?$

- α**-conversion**, written $=_\alpha$:
  conflict-free **renaming of bound names**
  (no new name-bindings shall be generated)

- **substitution** $[\vec{b}/\vec{a}]P$ (for matching $\vec{b}$ and $\vec{a}$, where $\vec{a}$ p.w.d.)
  replaces *all* **free** occurrences of a_i in P by b_i,
  possibly enforcing α-conversion.

Examples

$$
\begin{aligned}
(\boldsymbol{\nu} a)\,(\overline{a}.\mathbf{0} | b.\mathbf{0}) &=_\alpha (\boldsymbol{\nu} c)\,(\overline{c}.\mathbf{0} | b.\mathbf{0}) \\
&=_\alpha (\boldsymbol{\nu} b)\,(\overline{b}.\mathbf{0} | b.\mathbf{0})
\end{aligned}
$$

$$
\begin{aligned}
[a/b](\ (\boldsymbol{\nu} b)\,\overline{b}.\mathbf{0} \mid b.\mathbf{0}\ ) &=_\alpha (\ (\boldsymbol{\nu} b)\,\overline{a}.\mathbf{0} \mid a.\mathbf{0}\ ) \\
&=_\alpha (\ (\boldsymbol{\nu} b)\,\overline{b}.\mathbf{0} \mid a.\mathbf{0}\ )
\end{aligned}
$$

$$
\begin{aligned}
[a/b](\ (\boldsymbol{\nu} a)\,\overline{b}.a.\mathbf{0} \mid b.\mathbf{0}\ ) &=_\alpha (\ (\boldsymbol{\nu} a)\,\overline{a}.a.\mathbf{0} \mid a.\mathbf{0}\ ) \\
&=_\alpha (\ (\boldsymbol{\nu} c)\,\overline{a}.c.\mathbf{0} \mid a.\mathbf{0}\ )
\end{aligned}
$$

$$\ldots \qquad \text{PAR}_1\colon \frac{P_1 \xrightarrow{\mu} P_1'}{P_1|P_2 \xrightarrow{\mu} P_1'|P_2} \qquad \text{PAR}_2\colon \frac{P_2 \xrightarrow{\mu} P_2'}{P_1|P_2 \xrightarrow{\mu} P_1|P_2'}$$

$$\text{REACT}\colon \frac{P \xrightarrow{\lambda} P' \qquad Q \xrightarrow{\overline{\lambda}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\text{RES}\colon \frac{P \xrightarrow{\mu} P'}{(\boldsymbol{\nu} a)\, P \xrightarrow{\mu} (\boldsymbol{\nu} a)\, P'} \quad \text{IF } \mu \notin \{a, \overline{a}\}$$

$$\text{ALPHA}\colon \frac{Q \xrightarrow{\mu} Q'}{P \xrightarrow{\mu} P'} \quad \text{IF } P =_\alpha Q \text{ AND } P' =_\alpha Q'$$

Buffers, revisited . . .

$$\begin{array}{rcl} \mathcal{N} & := & \{\, \mathsf{in}_i, \mathsf{out}_i, \mathsf{x}_i \mid i \in \{0,1\} \,\} \\ \vec{a} & := & \mathsf{in}_0, \mathsf{in}_1, \mathsf{out}_0, \mathsf{out}_1 \\ \mathsf{Bluff}^{(2)}(\,\vec{a}\,) & \stackrel{\text{def}}{=} & (\boldsymbol{\nu}\mathsf{x}_0, \mathsf{x}_1)\ (\ \mathsf{Buff}^{(1)}\langle\, \mathsf{in}_0, \mathsf{in}_1, \mathsf{x}_0, \mathsf{x}_1 \,\rangle \\ & & \qquad\qquad \mid\ \mathsf{Buff}^{(1)}\langle\, \mathsf{x}_0, \mathsf{x}_1, \mathsf{out}_0, \mathsf{out}_1 \,\rangle\ ) \end{array}$$

- compare the behavior (= LTSs) of $\mathsf{Buff}^{(2)}$ and $\mathsf{Bluff}^{(2)}$
- regard both as lack boxes with "buttons" $\vec{a}$ . . .