Concurrency: Theory, Languages and Programming

– Equivalences for CCS –

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Bisimulation on CCS

check out Session 4, again

add 1+1 . . .

“Algebraic” Properties (I)

M M

–

– –

. . .

Why *algebraic* ?

“Algebraic” Properties (II)

For all , $\sum$.

For all and :

$\sum$

$\{$

$\sum$

—

“Algebraic” Properties (III)

For all , , and :

$\sum$

and $-$

$\{$

$\sum$

$-$

Expansion Law ! (also called: *Interleaving*)
Compare to the notions of *standard forms* in Milner’s book:
every process term can be transformed into a form
that matches the left-hand side of the above equation.

Process Contexts

<u>**Definition:**</u>A **process context** is (precisely) defined by the following syntax:

M M

The **elementary contexts** are

M, M , , , .

denotes the result of filling the hole of with process .

Process congruence

Definition: (Process congruence)
Let be an *equivalence relation* over .

Then is said to be a ***process congruence***, if
for *all* contexts ,
implies .

Process congruence (II)

Proposition:

An arbitrary equivalence relation is a process congruence if, and only if, it is preserved by all *elementary contexts*; i.e., if , then

$$M \qquad M$$

$$M \qquad M$$

Note:

For proving that an equivalence relation is a congruence, the elementary contexts suffice.

Congruence Properties

Proposition:

Bisimilarity is a process congruence, i.e., . . .

Towards Observation Equivalence

Let us assume that our LTSs may dispose of a single distinguished *internal action* symbol, say: , as is the case for our language of concurrent process expressions. Then:

“Different internal behavior” should “not count” !

Definition:(observations / weak actions)

1.

2.

Weak Simulation

Definition:

is a weak simulation **iff**, whenever ,

if then there is
such that and .

if then there is

such that and .

weakly simulates ,
if there is a weak simulation such that .

Example:

Prove that weakly simulates .
Prove that weakly simulates .

Weak Bisimulation

Definition:(* straightforward / should be no surprise *)
A binary relation is *a* **weak bisimulation**
if both and its converse are weak simulations.

and are **weakly bisimilar**, **weakly equivalent**, or **observation equivalent**, written ,
if there exists a weak bisimulation with .

Alternatively:

$\bigcup$ is weak bisimulation

Proposition:

1. is itself a weak bisimulation.
2. is an equivalence relation.

Strong vs Weak

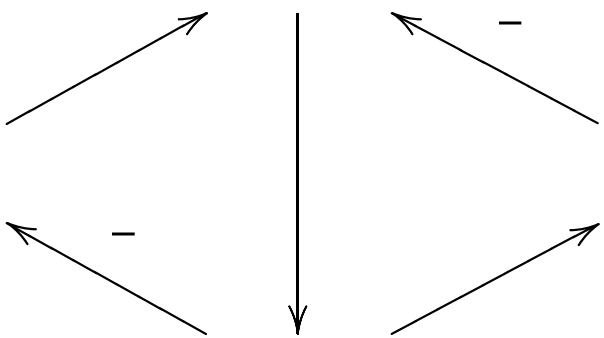
1. every strong simulation is also a weak one
2.   implies

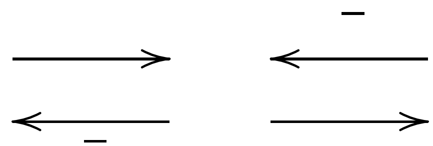
   Examples ?

   Proof ?

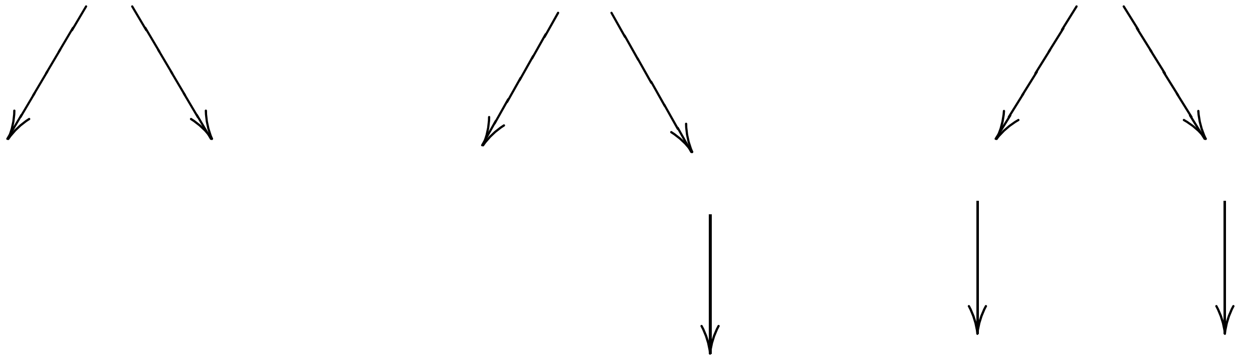
Example

Prove that .

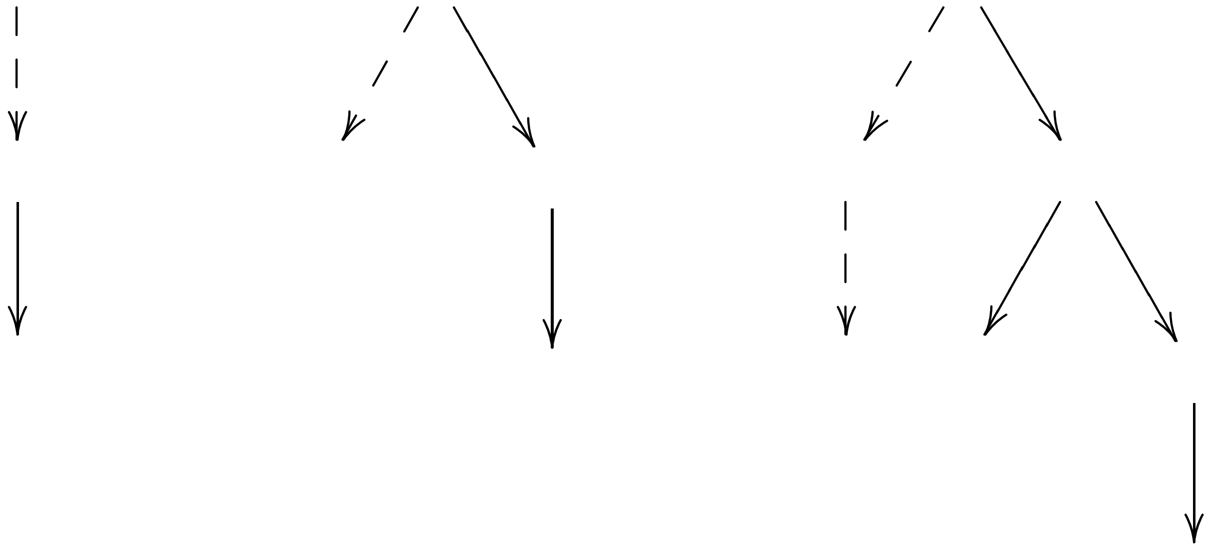




Some Inequivalences



Some Equivalences



Some Equations

Theorem:

Let be any process.

Let M any summations. Then:

1.
2. M M
3. M M

Congruence Properties

Proposition:

Weak bisimilarity is a process congruence, i.e., . . .

Example:

Observe !

Let .

Compare !

Two-Place Buffers

Buff	1-place buffer containing , where in out
Buff	in Buff
Buff	$\overline{\text{out}}$ Buff
Buff	2-place buffer containing — SPECIFICATION
Buff	in Buff
Buff	$\overline{\text{out}}$ Buff in Buff
Buff	$\overline{\text{out}}$ Buff
Bluff	2-place buffer containing — IMPLEMENTATION
Bluff	x Buff in x Buff x out

prove that Buff Bluff

Unique Solution of Equations

Theorem:

Let be a (possibly infinite) sequence of process variables. In the equations

assume that . Then, up to , there is a unique sequence of processes which satisfies the equations.