Data-flow analysis (II)

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(based on material by Michael Schwartzbach and Erik Stenman)

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Data-flow analyses

We have already seen how to use the data-flow analysis framework to compute an approximation of liveness for each program point.

We will now explore other kinds of information about programs which can be computed using that framework.

Data-flow analysis

Available expressions

Available expressions

A non-trivial expression in a program is **available** at some point if its value has already been computed earlier.

Data-flow analysis can be used to approximate the set of expressions available at all program points. The result from that analysis can then be used to eliminate common sub-expressions, for example.

Available expressions intuitions

We will compute the set of expressions available *after* every node of the CFG.

Intuitively, an expression *e* is available after some node *n* if it is available after all predecessors of *n*, or if it is defined by *n* itself, and not killed by *n*.

A node *n* kills an expression *e* if it gives a new value to a variable used by *e*. For example, the assignment x←y kills all expressions which use x, like x+1.

Available expressions equations

To approximate available expressions, we attach to every node n of the CFG a variable v_n containing the set of expressions available *after* it.

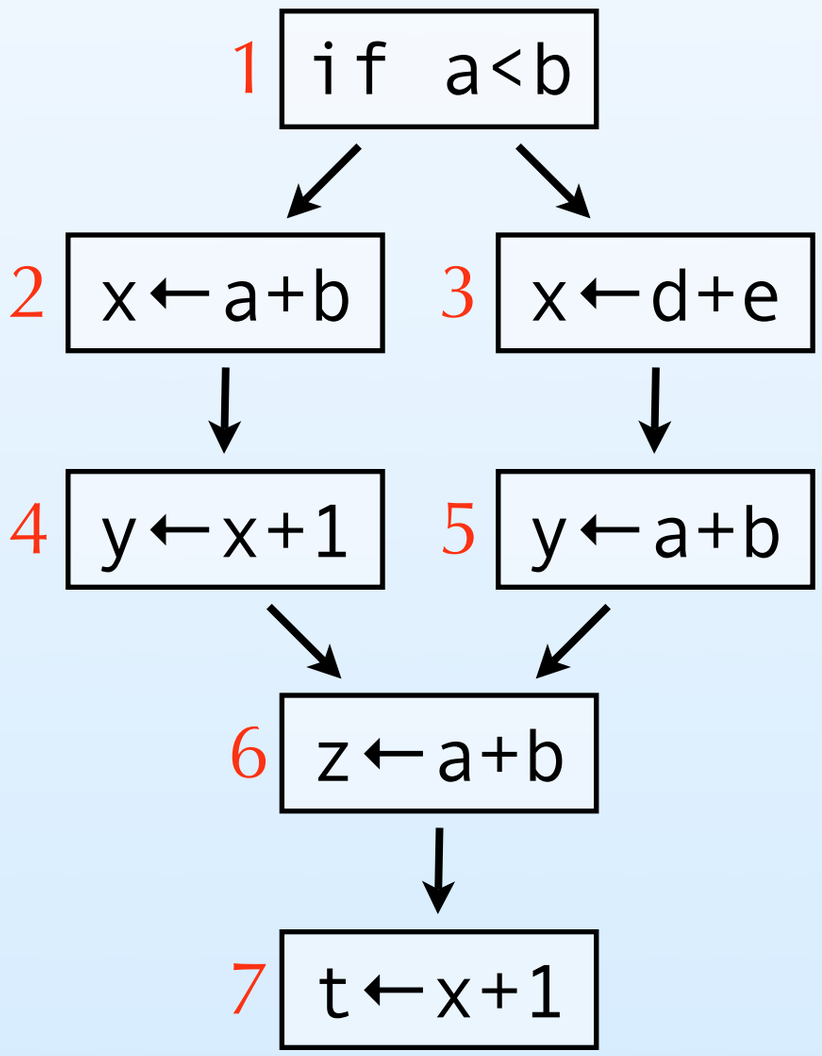
Then we derive constraints from the CFG nodes, which have the form:

$$v_n = (v_{p1} \cap v_{p2} \cap \ldots \setminus \text{kill}(n)) \cup \text{gen}(n)$$

where gen(n) is the set of expressions computed by n, and kill(n) the set of expressions killed by n.

Available expressions example

CFG



constraints

v_1={a<b}
v_2={a+b}∪(v_1↓x)
v_3={d+e}∪(v_1↓x)
v_4={x+1}∪(v_2↓y)
v_5={a+b}∪(v_3↓y)
v_6={a+b}∪(v_4∩v_5)↓z
v_7={x+1}∪v_5↓t

solution

v_1={a<b}
v_2={a+b, a<b}
v_3={d+e, a<b}
v_4={x+1, a+b, a<b}
v_5={a+b, d+e, a<b}
v_6={a+b, a<b}
v_7={x+1, a+b, a<b}

v_n=set of expressions live after node n.

Notation:
S↓x = S \ all expressions using variable x.

Data-flow analysis

Very busy expressions

Very busy expressions

An expression is **very busy** at some program point if it will definitely be evaluated before its value changes.

Data-flow analysis can approximate the set of very busy expressions for all program points. The result of that analysis can then be used to perform code hoisting: the computation of a very busy expression *e* can be performed at the earliest point where it is busy.

Very busy expressions intuitions

We will compute the set of very busy expressions before every node of the CFG.

Intuitively, an expression *e* is very busy before node *n* if it is evaluated by *n*, or if it is very busy in all successors of *n*, and it is not killed by *n*.

Very busy expressions equations

To approximate very busy expressions, we attach to node n of the CFG a variable v_n containing the set of expressions which are very busy *before* it.

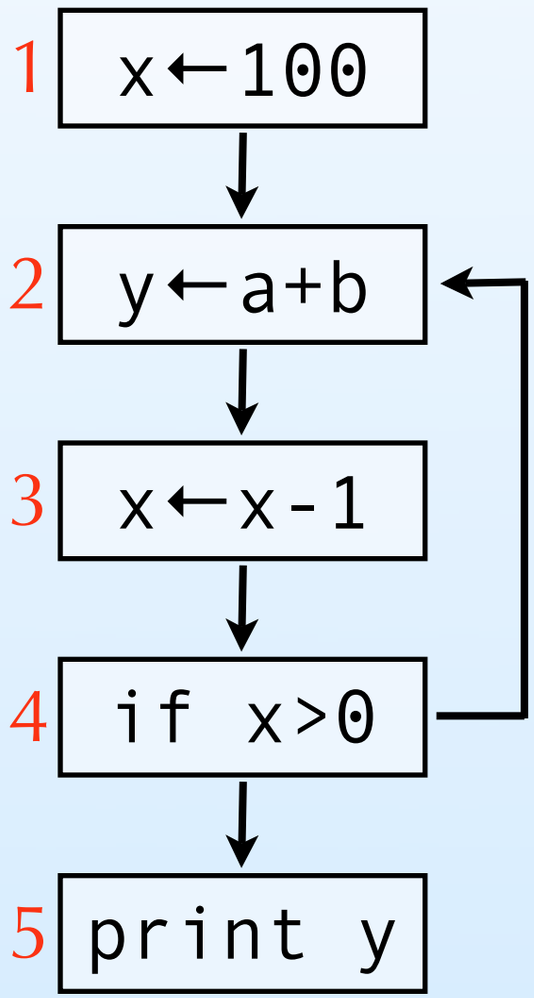
Then we derive constraints from the CFG nodes, which have the form:

$$v_n = (v_{s1} \cap v_{s2} \cap \ldots \setminus \text{kill}(n)) \cup \text{gen}(n)$$

where gen(n) is the set of expressions computed by n, and kill(n) the set of expressions killed by n.

Very busy expressions example

CFG



constraints

$v_1 = v_2 \downarrow x$

$v_2 = \{a+b\} \cup v_3 \downarrow y$

$v_3 = \{x-1\} \cup v_4 \downarrow x$

$v_4 = \{x>0\} \cup (v_5 \cap v_2)$

$v_5 = \{\}$

solution

$v_1 = \{a+b\}$

$v_2 = \{a+b, x-1\}$

$v_3 = \{x-1\}$

$v_4 = \{x>0\}$

$v_5 = \{\}$

v_n=set of expressions very busy before node *n*.

Notation:

$S \downarrow x$ = S \ all expressions using variable x.

Data-flow analysis

Reaching definitions

Reaching definitions

The **reaching definitions** for a program point are the assignments that may have defined the values of variables at that point.

Data-flow analysis can approximate the set of reaching definitions for all program points. These sets can then be used to perform constant propagation, for example.

Reaching definitions intuitions

We will compute the set of reaching definitions after every node of the CFG. That set will be represented as a set of CFG node identifiers.

Intuitively, the reaching definitions after a node *n* are all the reaching definitions of the predecessors of *n*, minus those which define a variable defined by *n* itself, plus *n* itself.

Reaching definitions equations (1)

To approximate reaching definitions, we attach to node *n* of the CFG a variable v_n containing the set of definitions (CFG nodes) which can reach *n*.

For a node *n* which is not an assignment, the reaching definitions are simply those of its predecessors:

$$v_n = (v_{p1} \cup v_{p2} \cup \ldots)$$

Reaching definitions equations (2)

For a node *n* which is an assignment, the equation is more complicated:

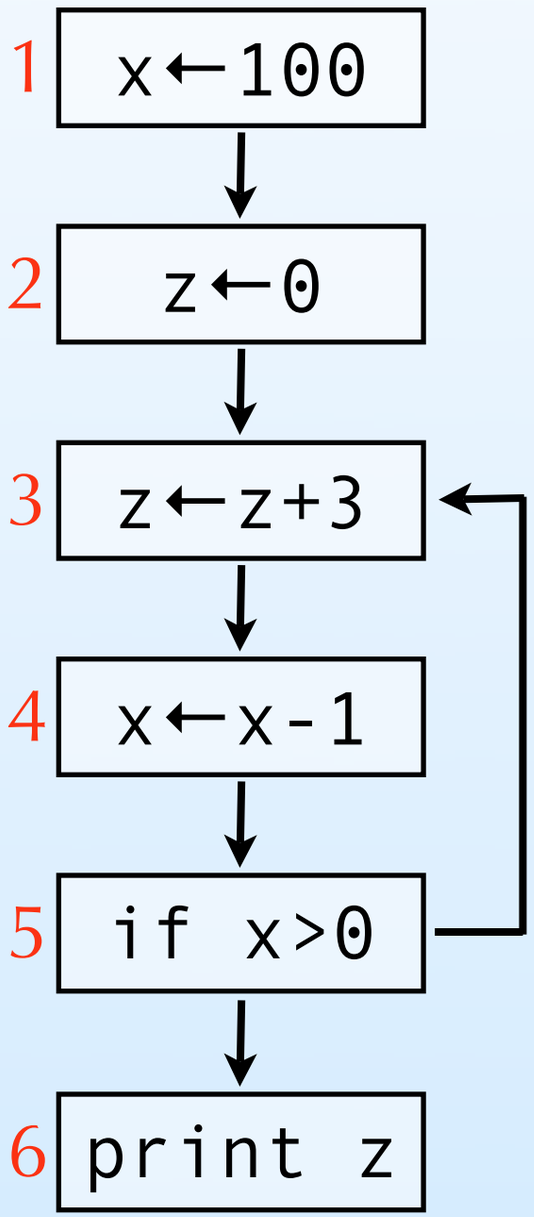
$$v_n = (v_{p1} \cup v_{p2} \cup \ldots) \setminus \text{kill}(n) \cup \{ n \}$$

where kill(*n*) are the definitions killed by *n*, *i.e.* those which define the same variable as *n* itself.

For example, a definition like x←y kills all expressions of the form x←...

Reaching definitions example

CFG



constraints

$v_1=\{1\}$

$v_2=v_1{\downarrow}z \cup \{2\}$

$v_3=(v_2 \cup v_5){\downarrow}z \cup \{3\}$

$v_4=v_3{\downarrow}x \cup \{4\}$

$v_5=v_4$

$v_6=v_5$

solution

$v_1=\{1\}$

$v_2=\{1,2\}$

$v_3=\{1,3,4\}$

$v_4=\{3,4\}$

$v_5=\{3,4\}$

$v_6=\{3,4\}$

v_n=set of reaching definitions after node n.

Notation:
$S{\downarrow}x = S \setminus$ all nodes defining variable x.

Using the result of data-flow analyses

Using data-flow analysis

Once a particular data-flow analysis has been conducted, its result can be used to optimise the analysed program.

We will quickly examine some transformations which can be performed using the data-flow analysis presented before.

Dead-code elimination

Useless assignments can be eliminated using liveness analysis, as follows:

> Whenever a CFG node *n* is of the form x←e, and x is not live after *n*, then the assignment is useless and node *n* can be removed.

Common sub-expression elimination

Common sub-expressions can be eliminated using availability information, as follows:

Whenever a CFG node *n* computes an expression of the form x *op* y and x *op* y is available before *n*, then the computation within *n* can be replaced by a reference to the previously-computed value.

Constant propagation

Constant propagation can be performed using the result of reaching definitions analysis, as follows:

When a CFG node *n* uses a value x and the only definition of x reaching *n* has the form x←c where c is a constant, then the use of x in *n* can be replaced by c.

Copy propagation

Copy propagation – very similar to constant propagation – can be performed using the result of reaching definitions analysis, as follows:

When a CFG node *n* uses a value x, and the only definition of x reaching *n* has the form x←y where y is a variable, and y is not redefined on any path leading to *n*, then the use of x in *n* can be replaced by y.

Register allocation

To assign machine registers to program variables, liveness analysis is required.

We will explore register allocation in more details later, but intuitively it should be clear that a set of variables $V=\{ v_1, v_2, \ldots, v_n \}$ can be allocated to a single machine register provided that no two variables in V are live simultaneously.

Summary

Data-flow analysis is a framework that can be used to approximate various properties about programs.

We have seen how to use the data-flow analysis framework to approximate liveness, available expressions, very busy expressions and reaching definitions. The result of those analysis can be used to perform various optimisations like dead-code elimination, constant propagation, etc.