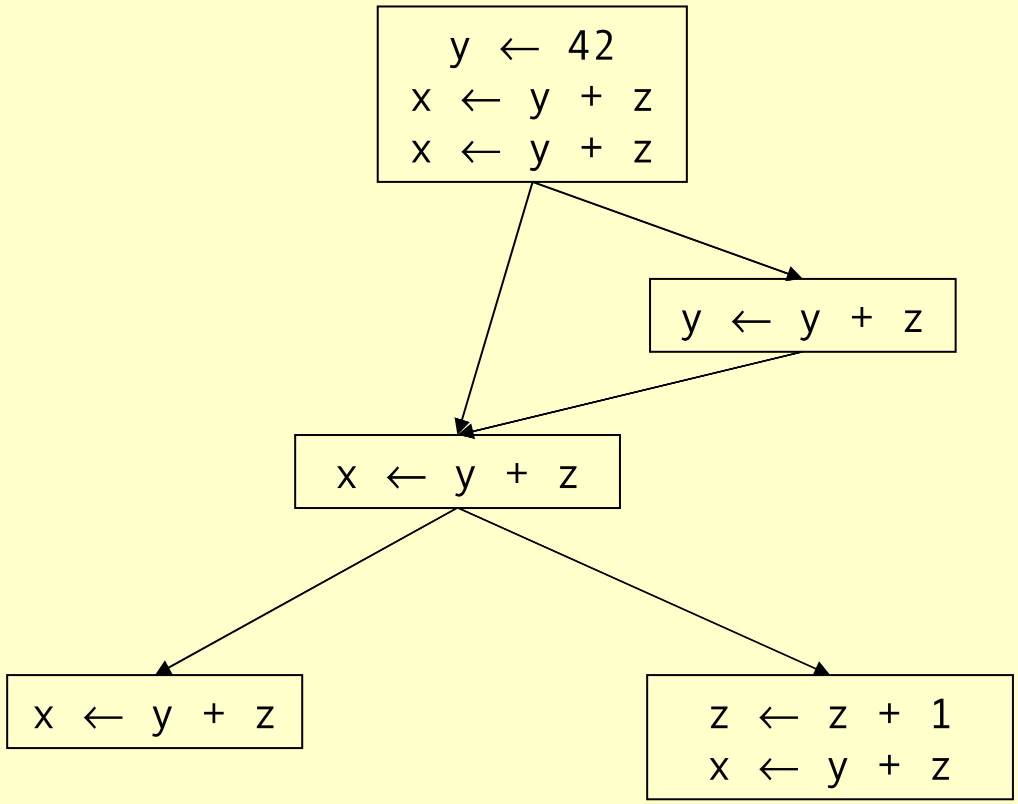
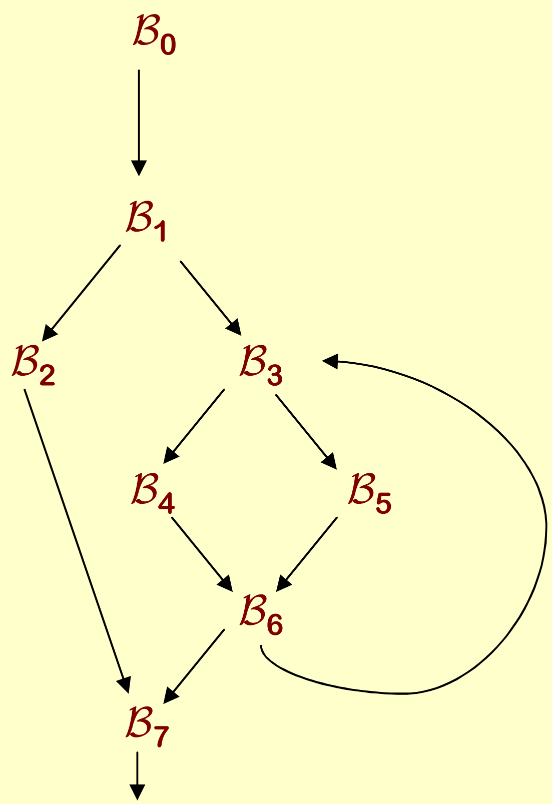
Exercises

E1: Find all redundant, and partially redundant expressions in this CFG, and optimize the code.



E2: Find the Loop



- Find edges whose heads (>) **dominate** tails (-), these edges are back edges of loops.
- Given a back edge n→d:
  - The node d is the loop header.
  - The loop consists of n plus all nodes that can reach n without going through d (all nodes "between" d and n)

loop(d,n)
    loop = {d}; stack = ∅; *insert*(n);
    while stack not empty do
        m = pop stack;
        for all p ∈ *pred*(m) do *insert*(p);
insert(m)
    if m ∉ loop then
        loop = loop ∪ {m};
        push m onto stack;

E3: Write java-pseudo code for finding dominators.

- A node $\mathcal{N}$ dominates $\mathcal{M}$ iff $\mathcal{N}$ is on every path from $\mathcal{N}_0$ to $\mathcal{M}$
  - Every node dominates itself.
  - $\mathcal{N}$'s immediate dominator is its closest dominator, IDOM($\mathcal{N}$)
  - Initially: DOM(n) = $\mathbb{N}$, $\forall\ n \neq n_0$

$\text{DOM}(\mathcal{N}_0) = \{\mathcal{N}_0\}$

$\text{DOM}(\mathcal{N}) = \{\mathcal{N}\} \cup \left(\bigcap_{\mathcal{P} \in preds(\mathcal{N})} \text{DOM}(\mathcal{P})\right)$

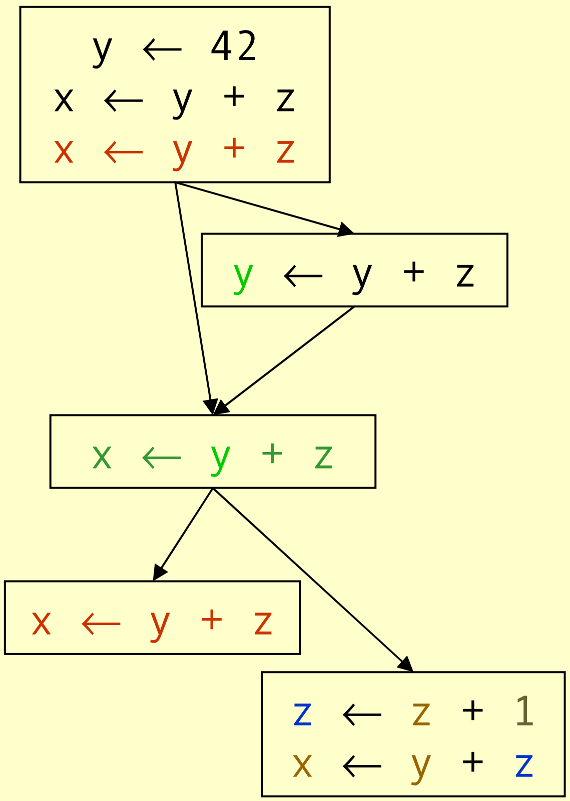
E4: Write Java-pseudo code to find loops in the CFG.

Find backedges whose heads **dominate** tails.
Given a back edge n→d: call loop(d,n)

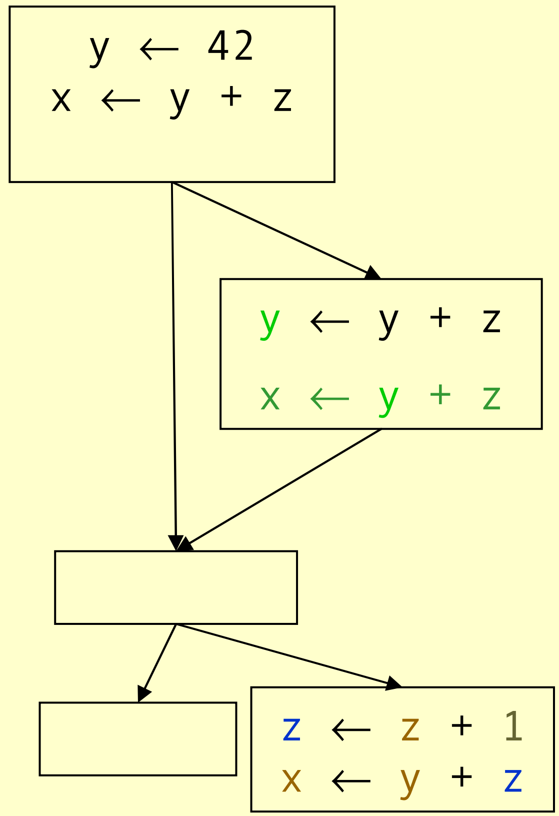
```
loop(d,n)
    loop = {d}; stack = ∅; insert(n);
    while stack not empty do
        m = pop stack;
        for all p ∈ pred(m) do insert(p);
insert(m)
    if m ∉ loop then
        loop = loop ∪ {m};
        push m onto stack;
```

Solutions

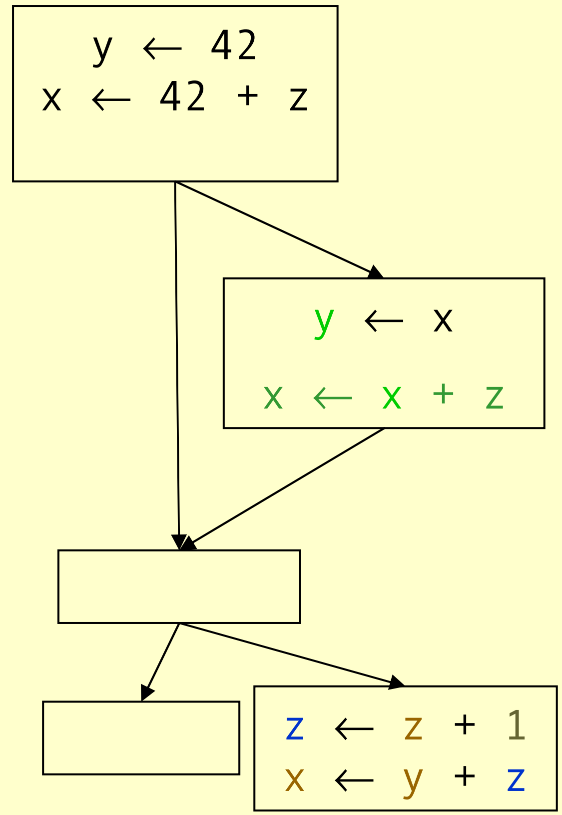
S1: Find all redundant, and partially redundant expressions in this CFG.



Redundant expr.



Available expr, copy prop, & constant prop.



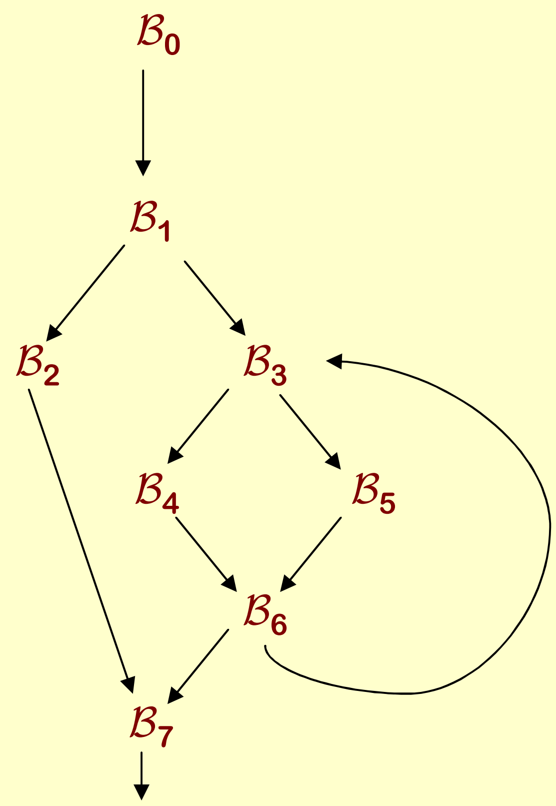
Compute Dominators

- A node $\mathcal{N}$ dominates $\mathcal{M}$ iff $\mathcal{N}$ is on every path from $\mathcal{N}_0$ to $\mathcal{M}$
  - Every node dominates itself.
  - $\mathcal{N}$'s <u>immediate dominator</u> is its closest dominator, IDOM($\mathcal{N}$)
  - Initially: DOM(n) = $\mathbb{N}$, $\forall\ n \neq n_0$

$$\text{DOM}(\mathcal{N}_0) = \{\mathcal{N}_0\}$$

$$\text{DOM}(\mathcal{N}) = \{\mathcal{N}\} \cup \left(\bigcap_{\mathcal{P} \in preds(\mathcal{N})} \text{DOM}(\mathcal{P})\right)$$

Compute Dominators



Initial

n	DOM(n)	Preds(n)
B_0	$\{B_0\}$	$\{\}$
B_1	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_0\}$
B_2	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_1\}$
B_3	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_1, B_6\}$
B_4	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_5	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_6	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_4, B_5\}$
B_7	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_2, B_6\}$

$$\text{DOM}(\mathcal{B}_1) = \{\mathcal{B}_1\} \cup \left(\cap_{\mathcal{P} \in preds(\mathcal{B}_1)} \text{DOM}(\mathcal{P})\right)$$

$$= \{\mathcal{B}_1\} \cup (\{\mathcal{B}_0\})$$

$$= \{B_1, B_0\}$$

n	DOM(n)	Preds(n)
B_0	$\{B_0\}$	{}
B_1	**$\{B_0, B_1\}$**	$\{B_0\}$
B_2	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_1\}$
B_3	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_1, B_6\}$
B_4	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_5	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_6	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_4, B_5\}$
B_7	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_2, B_6\}$

$$\text{DOM}(n) = \{n\} \cup \left(\bigcap_{p \in preds(n)} \text{DOM}(p)\right)$$

n	DOM(n)	Preds(n)
B_0	$\{B_0\}$	$\{\}$
B_1	$\{B_0, B_1\}$	$\{B_0\}$
$\mathbf{B_2}$	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{\mathbf{B_1}\}$
B_3	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_1, B_6\}$
B_4	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_5	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_6	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_4, B_5\}$
B_7	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_2, B_6\}$

$\text{DOM}(n) = \{n\} \cup (\cap_{\mathcal{P} \in preds(n)} \text{DOM}(p))$

n	DOM(n)	Preds(n)
B_0	$\{B_0\}$	$\{\}$
B_1	$\{B_0, B_1\}$	$\{B_0\}$
B_2	$\{\mathbf{B_0}, \mathbf{B_1}, \mathbf{B_2}\}$	$\{B_1\}$
B_3	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_1, B_6\}$
B_4	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_5	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_6	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_4, B_5\}$
B_7	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_2, B_6\}$

$$\text{DOM}(B_3) = \{B_3\} \cup (\text{DOM}(B_1) \cap \text{DOM}(B_6))$$
$$= \{B_3\} \cup (\{B_0, B_1\} \cap \{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\})$$
$$= \{B_0, B_1, B_3\}$$

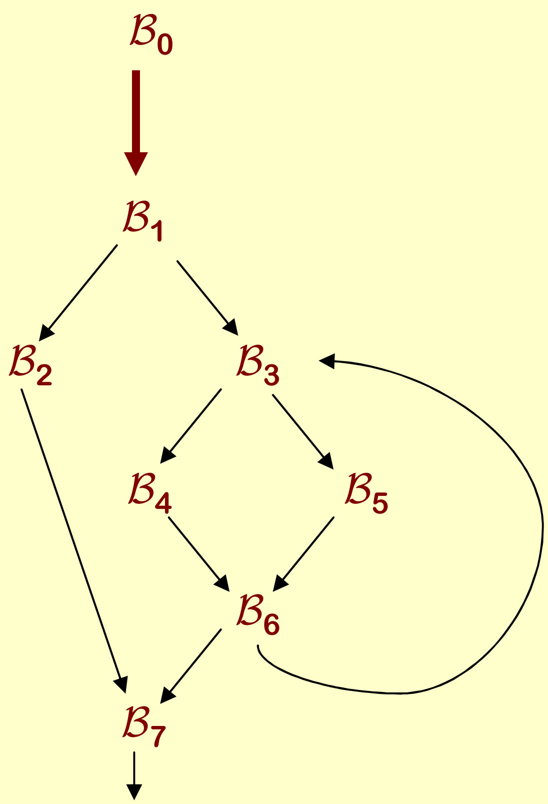
n	DOM(n)	Preds(n)
B_0	$\{B_0\}$	{}
B_1	$\{B_0, B_1\}$	$\{B_0\}$
B_2	$\{B_0, B_1, B_2\}$	$\{B_1\}$
$\mathbf{B_3}$	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{\mathbf{B_1}, \mathbf{B_6}\}$
B_4	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_5	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_3\}$
B_6	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_4, B_5\}$
B_7	$\{B_0, B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$	$\{B_2, B_6\}$

$$\text{DOM}(n) = \{n\} \cup \left(\bigcap_{\mathcal{P} \in \, preds(n)} \text{DOM}(p)\right)$$

n	DOM(n)	Preds(n)
B_0	$\{B_0\}$	$\{\}$
B_1	$\{B_0, B_1\}$	$\{B_0\}$
B_2	$\{B_0, B_1, B_2\}$	$\{B_1\}$
B_3	$\{B_0, B_1, B_3\}$	$\{B_1, B_6\}$
B_4	$\{B_0, B_1, B_3, B_4\}$	$\{B_3\}$
B_5	$\{B_0, B_1, B_3, B_5\}$	$\{B_3\}$
B_6	$\{B_0, B_1, B_3, B_6\}$	$\{B_4, B_5\}$
B_7	$\{B_0, B_1, B_7\}$	$\{B_2, B_6\}$

Find the Loop

- Find edges whose heads (>) dominate tails (-), these edges are back edges of loops.



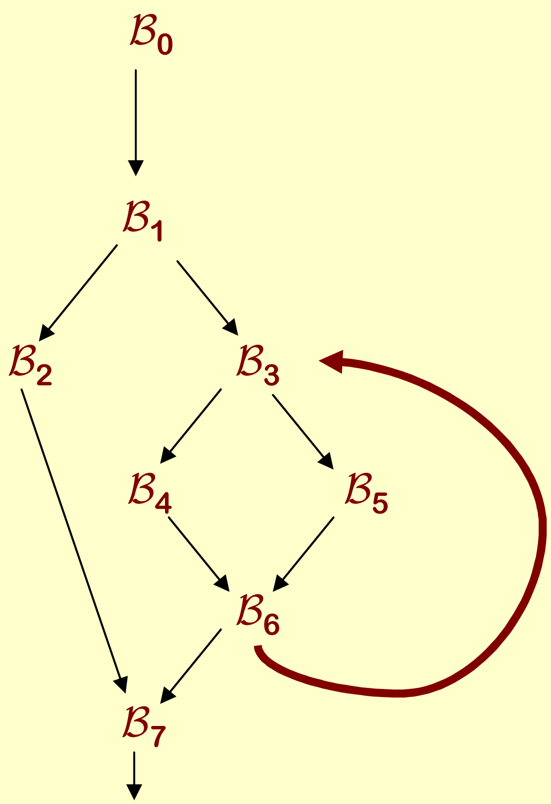
Edges

$B_{0\text{->}}B_1$
B_1->B_2
B_1->B_3
B_2->B_7
B_3->B_4
B_3->B_5
B_4->B_6
B_5->B_6
B_6->B_3
B_6->B_7

n	DOM(n)
$\mathbf{B_0}$	$\{B_0\}$
B_1	$\{B_0, B_1\}$
B_2	$\{B_0, B_1, B_2\}$
B_3	$\{B_0, B_1, B_3\}$
B_4	$\{B_0, B_1, B_3, B_4\}$
B_5	$\{B_0, B_1, B_3, B_5\}$
B_6	$\{B_0, B_1, B_3, B_6\}$
B_7	$\{B_0, B_1, B_7\}$

Find the back edges

- Find edges whose heads (>) dominate tails (-), these edges are back edges of loops.



Edges

$B_{0\text{->}}B_1$
B_1->B_2
B_1->B_3
B_2->B_7
B_3->B_4
B_3->B_5
B_4->B_6
B_5->B_6
B_6->B_3
B_6->B_7

n	DOM(n)
B_0	$\{B_0\}$
B_1	$\{B_0, B_1\}$
B_2	$\{B_0, B_1, B_2\}$
B_3	$\{B_0, B_1, B_3\}$
B_4	$\{B_0, B_1, B_3, B_4\}$
B_5	$\{B_0, B_1, B_3, B_5\}$
B_6	$\{B_0, B_1, \mathbf{B_3}, B_6\}$
B_7	$\{B_0, B_1, B_7\}$

Find the Loop

```
loop(d,n)
    loop = {d}; stack = ∅; insert(n);
    while stack not empty do
        m = pop stack;
        for all p ∈ pred(m) do insert(p);
insert(q)
    if q ∉ loop then
        loop = loop ∪ {q};
        push q onto stack;
```

$n = B_6$, $d = B_3$

$loop = \{B_3, B_6\}$

$stack = \{B_6\}$

n	Preds(n)
B_0	$\{\}$
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
    m = pop stack;
    for all p ∈ pred(m) do
        insert(p);
insert(q)
    if q ∉ loop then
        loop = loop ∪ {q};
        push q onto stack;
```

$n = B_6$, $d = B_3$

$loop = \{B_3, B_6\}$

stack = {**B_6**}

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
   m = pop stack;
   for all p ∈ pred(m) do
      insert(p);
insert(q)
   if q ∉ loop then
         loop = loop ∪ {q};
         push q onto stack;
```

$n = B_6$, $d = B_3$

$loop = \{B_3 , B_6\}$

stack = {}

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{\mathbf{B_4, B_5}\}$
B_7	$\{B_2, B_6\}$

while stack != ∅
  m = pop stack;
  for all $p \in$ ***pred*****(m)** do
    insert(p);
insert(q)
  if $q \notin$ loop then
    $loop = loop \cup \{q\}$;
    push q onto stack;

$n = B_6$, $d = B_3$

$loop = \{B_3 , B_6\}$

stack = {}

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{\mathbf{B_4, B_5}\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
   m = pop stack;
   for all p ∈ pred(m) do
      insert(p);
insert(q)
   if q ∉ loop then
      loop = loop ∪ {q};
      push q onto stack;
```

$n = B_6$, $d = B_3$

loop = $\{B_3, B_4, B_6\}$ stack = $\{B_4\}$

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
$\mathbf{B_5}$	$\{B_3\}$
B_6	$\{\mathbf{B_4}, \mathbf{B_5}\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
    m = pop stack;
    for all p ∈ pred(m) do
        insert(p);
insert(q)
    if q ∉ loop then
        loop = loop ∪ {q};
        push q onto stack;
```

$n = B_6$, $d = B_3$

loop = $\{B_3, B_4, B_5, B_6\}$ stack = $\{B_4, B_5\}$

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{\mathbf{B_4}, \mathbf{B_5}\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
    m = pop stack;
    for all p ∈ pred(m) do
        insert(p);
insert(q)
    if q ∉ loop then
        loop = loop ∪ {q};
        push q onto stack;
```

$n = B_6$, $d = B_3$

loop = $\{B_3, B_4, B_5, B_6\}$ stack = $\{$**B_4**, $B_5\}$

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
    m = pop stack;
    for all p ∈ pred(m) do
        insert(p);
insert(q)
    if q ∉ loop then
        loop = loop ∪ {q};
        push q onto stack;
```

$n = B_6$, $d = B_3$

loop = $\{B_3, B_4, B_5, B_6\}$ stack = $\{B_5\}$

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{\mathbf{B_3}\}$
B_5	$\{B_3\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

```
while stack != ∅
   m = pop stack;
   for all p ∈ pred(m) do
      insert(p);
insert(q)
   if q ∉ loop then
         loop = loop ∪ {q};
         push q onto stack;
```

$n = B_6$, $d = B_3$

loop = $\{B_3, B_4, B_5, B_6\}$ stack = $\{B_5\}$

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{$**B_3**$\}$
B_5	$\{B_3\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

while stack != $\emptyset$
    m = pop stack;
    for all p $\in$ *pred*(m) do
        insert(p);

insert(**q**)
    if **q** $\notin$ **loop** then
        loop = loop $\cup$ {q};
        push q onto stack;

$n = B_6$, $d = B_3$

loop = $\{B_3, B_4, B_5, B_6\}$ stack = {}

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{\mathbf{B_3}\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

while stack != $\varnothing$
m = pop stack;
for all p $\in$ *pred*(m) do
insert(p);

insert(**q**)
if **q** $\notin$ **loop** then
loop = loop $\cup$ {q};
push q onto stack;

$n = B_6$, $d = B_3$

$loop = \{B_3, B_4, B_5, B_6\}$ $stack = \{\}$

n	Preds(n)
B_0	{}
B_1	$\{B_0\}$
B_2	$\{B_1\}$
B_3	$\{B_1, B_6\}$
B_4	$\{B_3\}$
B_5	$\{B_3\}$
B_6	$\{B_4, B_5\}$
B_7	$\{B_2, B_6\}$

while **stack != ∅**
  m = pop stack;
  for all p ∈ *pred*(m) do
    insert(p);
insert(q)
  if q ∉ loop then
    loop = loop ∪ {q};
    push q onto stack;