Using Program Analysis for Optimization

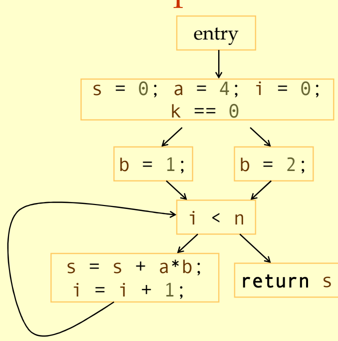
This lecture is primarily based on Konstantinos Sagonas set of slides (**Advanced Compiler Techniques**, (2AD518) at Uppsala University, January-February 2004). Used with kind permission.

Analysis and Optimizations

- Program Analysis
  - Discover properties of a program.
- Optimizations
  - Use analysis results to transform the program.
  - Goal: improve some aspect of the program
    - number of executed instructions, number of cycles
    - cache hit rate
    - memory space (code or data)
    - power consumption
  - Has to be safe: Keep the semantics of the program.

Control Flow Graph

```
int add(n, k) {
  s = 0; a = 4; i = 0;
  if (k == 0)
      b = 1;
  else
      b = 2;
  while (i < n) {
      s = s + a*b;
      i = i + 1;
  }
  return s;
}
```



Control Flow Graph

- Nodes represent computation.
  - Each node is a Basic Block (BB).
  - Basic Block is a sequence of instructions with:
    - No branches out of middle of basic block.
    - No branches into middle of basic block.
    - Basic blocks should be maximal.
  - Execution of basic block starts with first instruction.
  - Includes all instructions in basic block.
- Edges represent control flow.

Two Kinds of Variables

- Temporaries (temps, a tmp):
  - Introduced by the compiler.
  - Transfer values only within basic block.
  - Introduced as part of instruction flattening.
  - Introduced by optimizations/transformations.
- Program variables (vars, a var):
  - Declared in original program.
  - May transfer values between basic blocks.

Basic Block Optimizations
(Local Optimizations)

- Common Sub-Expression Elimination (CSE)
  `a=(x+y)+z; b=x+y;`
  `t=x+y; a=t+z; b=t;`
- Constant Propagation
  `x=5; b=x+y;`
  `b=5+y;`
- Algebraic Simplification
  `a=x*1;`
  `a=x;`
- Copy Propagation
  `a=x+y; b=a; c=b+z;`
  `a=x+y; b=a; c=a+z;`
- Dead Code Elimination
  `a=x+y; b=a; c=a+z;`
  `a=x+y; c=a+z`
- Strength Reduction
  `t=i*4;`
  `t=i<<2;`

Value Numbering

- Normalize BB so that all statements are of the form:
  - var = var op var (where op is a binary operator)
  - var = op var (where op is a unary operator)
  - var = var
  (I.E., no complex statements like x=a+b*c.)
- Simulate execution of basic block:
  - Assign a virtual value to each variable.
  - Assign a virtual value to each expression.
  - Assign a temporary variable to hold value of each computed expression.

Value Numbering for CSE

As we simulate execution of program, generate a new version of program:

- Each new value assigned to temporary
  a=x+y; becomes
  a=x+y; t_1=a;
- Temporary preserves value for use later in program even if original variable rewritten
  a=x+y; a=a+z; becomes
  a=x+y; t_1=a; a=a+z; t_2=a;

CSE Example

- Original
  a=x+y
  b=a+z
  b=b+y
  c=a+z
- After CSE
  a=x+y
  t_1=a
  b=a+z
  t_2=b
  b=b+y
  t_3=b
  c=t_2
- Issues:
  - CSE with different names:
    a=x; b=x+y; c=a+y;
  - Excessive temp generation and use.

Original Basic Block
a=x+y
b=a+z
b=b+y
c=a+z

New Basic Block
a=x+y
t_1=a
b=a+z
t_2=b
b=b+y
t_3=b
c=t_2

Var to Val
x→v_1
y→v_2
a→v_3
z→v_4
b→v_5 b→v_6
c→v_5

Exp to Val
v_1+v_2→v_3
v_3+v_4→v_5
v_5+v_2→v_6

Exp to Tmp
v_1+v_2→t_1
v_3+v_4→t_2
v_5+v_2→t_3

Problems

- Algorithm has a temporary for each value.
  a=x+y; t_1=a;
- Introduces
  - lots of temporaries.
  - lots of copy statements to temporaries.
- In many cases, temporaries and copy statements are unnecessary.
- So we eliminate them with copy propagation and dead code elimination.

Copy Propagation (CP)

- Once again, simulate execution of program
- If possible, use the original variable instead of a temporary
  - a=x+y; b=x+y;
  - After CSE becomes a=x+y; t_1=a; b=t_1;
  - After CP becomes a=x+y; b=a;
- **Key idea**: determine when original variables are **NOT** overwritten between computation of stored value and use of stored value.

Copy Propagation Maps

- Maintain two maps
  - tmp to var: tells which variable to use instead of a given temporary variable.
  - var to set: inverse of tmp to var. Tells which temps are mapped to a given variable by tmp to var.

BB Opt: Copy Propagation

Copy Propagation Example

- Original

```
a=x+y
b=a+z
c=x+y
a=b
```

- After CSE

```
a=x+y
t1=a
b=a+z
t2=b
c=t1
a=b
```

- After CSE and Copy Propagation

```
a=x+y
t1=a
b=a+z
t2=b
c=a
a=b
```

BB Opt: Copy Propagation

Copy Propagation Example

Basic Block After CSE	Basic Block After CSE and Copy Prop
a=x+y	a=x+y
t_1=a	t_1=a
b=a+z	b=a+z
t_2=b	t_2=b
c=t_1	c=a
a=b	a=b
tmp to var	**var to set**
$t_1 \rightarrow$ a	a $\rightarrow$ {t_1}
$t_2 \rightarrow$ b	b $\rightarrow$ {t_2}

BB Opt: Copy Propagation

Copy Propagation Example

Basic Block After CSE	Basic Block After CSE and Copy Prop
a=x+y	a=x+y
t_1=a	t_1=a
b=a+z	b=a+z
t_2=b	t_2=b
c=t_1	c=a
a=b	**a=b**
tmp to var	**var to set**
$t_1 \rightarrow t_1$	**a $\rightarrow$ {}**
$t_2 \rightarrow$ b	b $\rightarrow$ {t_2}

BB Opt: Copy Propagation

Dead Code Elimination

- Copy propagation keeps all temporaries.
- There may be temps that are never read.
- Dead Code Elimination removes them.

Basic block after CSE and Copy Prop.	Basic block after CSE, CP, & Dead Code Elimination
a=x+y	a=x+y
t1=a	
b=a+z	b=a+z
t2=b	
c=a	c=a
a=b	a=b

BB Opt: Dead Code Elimination

Dead Code Elimination

- Basic idea:
  - Process code in **reverse** execution order.
  - Maintain a set of variables that are needed later in computation.
  - On encountering an assignment to a temporary that is not needed, we remove the assignment.

BB Opt: Dead Code Elimination

Basic Block After CSE and Copy Prop	Needed Set
a=x+y	{a,z}
t1=a	{a,z}
b=a+z	{a,b,z}
t2=b	{a,b}
c=a	{a,b}
⇒ a=b	{b}

Interesting Properties

- Analysis and optimization algorithms simulate execution of the program.
  - CSE and Copy Propagation go forward.
  - Dead Code Elimination goes backwards.
- Optimizations are stacked.
  - Group of basic transformations.
  - Work together to get good result.
  - Often, one transformation creates inefficient code that is cleaned up by following transformations.

Other Basic Block Transformations

- Constant Propagation.
- Strength Reduction:
  - `a*4;` ⇒ `a<<2;`
  - `3*a;` ⇒ `a+a+a;`
- Algebraic Simplification:
  - `a*1;` ⇒ `a;`
  - `b+0;` ⇒ `b;`
- Unified transformation framework.

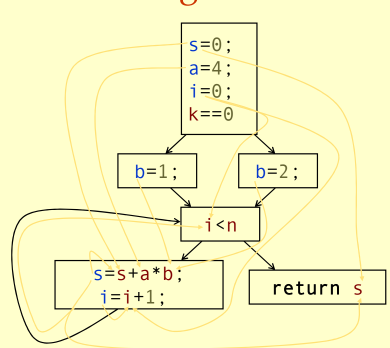
Dataflow Analysis
(Global Analysis)

- Used to determine properties of programs that involve multiple basic blocks.
- Typically used to enable transformations.
  - common sub-expression elimination.
  - constant and copy propagation.
  - dead code elimination.
- Analysis and transformation often come in pairs.

Reaching Definitions

- Concept of *definition* and *use*
  - `a=x+y`
    - is a definition of a.
    - is a use of x and y.
- A definition reaches a use if value written by definition may be read by use.

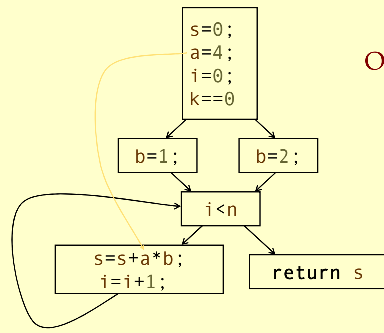
Reaching Definitions



Reaching Definitions and Constant Propagation

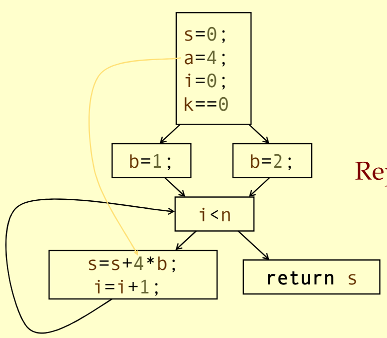
- Is a use of a variable a constant?
  - Check all reaching definitions.
  - If all assign variable to same constant.
  - Then use is in fact a constant.
- Can replace variable with constant.

Is a constant in s=s+a*b?



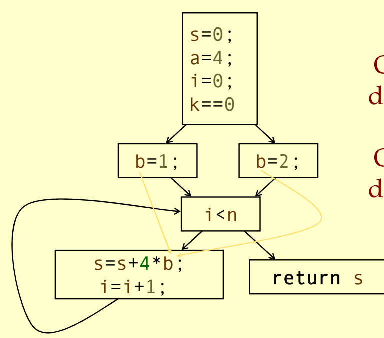
Yes!
On all reaching definitions
a=4

Constant Propagation Transform



Yes!
a=4
in
s=s+a*b
Replace use of a with 4.

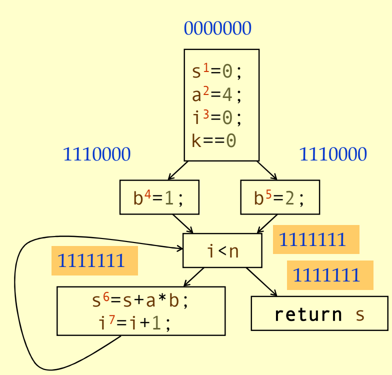
Is b constant in s=s+4*b?



No!
One reaching definition with
b=1
One reaching definition with
b=2

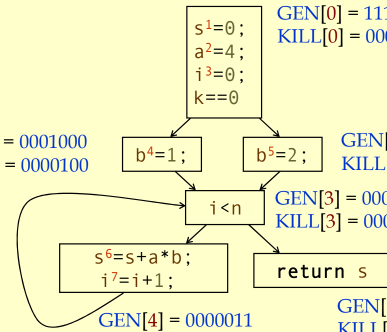
Computing Reaching Definitions

- Compute with sets of definitions:
  - Represent sets using bit vectors.
  - Each definition has a position in bit vector.
- At each basic block, compute:
  - Definitions that reach start of block.
  - Definitions that reach end of block.
- Do computation by simulating execution of program until the fixed point is reached.



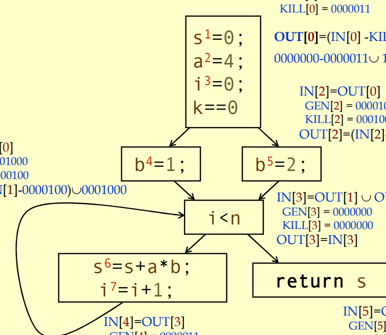
Formalizing Analysis

- Each basic block has
  - IN - set of definitions that reach beginning of block
  - OUT - set of definitions that reach end of block
  - GEN - set of definitions generated in block
  - KILL - set of definitions killed in the block
- GEN[`s⁶=s+a*b;i⁷=i+1;`] = 0000011
- KILL[`s⁶=s+a*b;i⁷=i+1;`] = 1010000
- Compiler scans each basic block to derive GEN and KILL sets.



Dataflow Equations

- $IN[b_i] = OUT[b_1] \cup \ldots \cup OUT[b_n]$
  where $b_1, \ldots, b_n$ are predecessors of b_i
- $OUT[b_i] = (IN[b_i] - KILL[b_i]) \cup GEN[b_i]$
- IN[entry] = 0000000
- Result: system of equations.



Solving Equations

- Use fix point algorithm.
- Initialize with solution of $OUT[b_i] = 0000000$
- Repeatedly apply equations:
  - $IN[b_i] = OUT[b_1] \cup \ldots \cup OUT[b_n]$
  - $OUT[b_i] = (IN[b_i] - KILL[b_i]) \cup GEN[b_i]$
- Until reach fixed point, i.e., until equation application has no further effect.
- Use a worklist to track which equation applications may have further effect.

Reaching Definitions Algorithm

```
for all nodes n∈ℕ
  OUT[n] = ∅;                            // Or OUT[n] = GEN[n];
Changed = ℕ;                             // ℕ = all nodes in graph
while (Changed != ∅)                     // Until fixed point reached.
  choose n∈Changed;                      // Node from worklist
  Changed=Changed-{n};                   // Remove from worklist
  OldOut = OUT[n]                        // Remember old result
  IN[n] = ∅;                             // Calculate IN as join
  for all nodes p∈predecessors(n)        //    of predecessors.
      IN[n]=IN[n]∪OUT[p];
  OUT[n]=(IN[n]-KILL[n])∪GEN[n];         // Recalculate OUT
  if (OUT[n] != OldOut)                  // If OUT[n] changed
   for all nodes s∈successors(n)
      Changed=Changed∪{s};               //Add succs to worklist
```

Questions

- Does the algorithm halt?
  - yes, because transfer function is monotonic.
  - if increase IN, increase OUT.
  - in limit, all bits are 1.
- If bit is 1, is there always an execution in which corresponding definition reaches basic block?
- If bit is 0, does the corresponding definition ever reach basic block?
- Concept of conservative analysis.

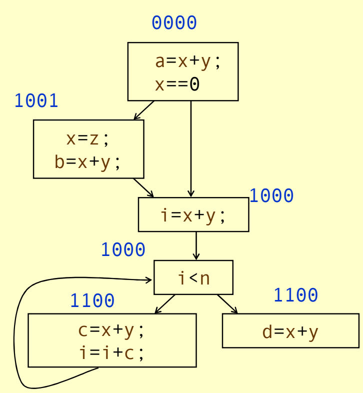
Available Expressions

- An expression x+y is available at a point p if
  - **every** path from the initial node to p evaluates x+y before reaching p,
  - and there are **no assignments** to x or y after the evaluation but before p.
- Available Expression information can be used to do global (across basic blocks) CSE.
- If an expression is available at use, there is no need to re-evaluate it.

Computing Available Expressions

- Represent sets of expressions using bit vectors.
- Each expression corresponds to a bit.
- Run dataflow algorithm similar to reaching definitions.
- Big difference:
  - Definition reaches a basic block if it comes from **ANY** predecessor in CFG.
  - Expression is available at a basic block only if it is available from **ALL** predecessors in CFG.

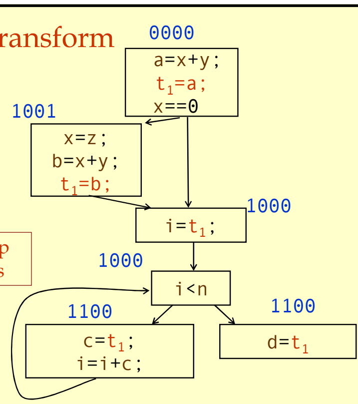
Expressions
1: x+y
2: i<n
3: i+c
4: x==0



Global CSE Transform

Expressions
1: x+y
2: i<n
3: i+c
4: x==0

Must use same temp for CSE in all blocks



Formalizing Analysis

- Each basic block has
  - IN - set of expressions that reach beginning of block.
  - OUT - set of expressions that reach end of block.
  - GEN - set of expressions generated in block.
  - KILL - set of expressions killed in the block.
- GEN[x=z; b=x+y] = 1000
- KILL[x=z; b=x+y] = 1001
- Compiler scans each basic block to derive GEN and KILL sets.

Dataflow Equations

- $IN[b_i] = OUT[b_1] \cap \ldots \cap OUT[b_n]$
  - where $b_1, \ldots, b_n$ are predecessors of b_i
- $OUT[b_i] = (IN[b_i] - KILL[b_i]) \cup GEN[b_i]$
- IN[entry] = 0000
- Result: system of equations.

Solving Equations

- Use fix point algorithm.
- IN[entry]=0000
- Initialize with solution of $OUT[b_i] = 1111$
- Repeatedly apply equations:
  - $IN[b_i] = OUT[b_1] \cap \ldots \cap OUT[b_n]$
  - $OUT[b_i] = (IN[b_i] - KILL[b_i]) \cup GEN[b_i]$
- Use a worklist to track which equation applications may have further effect.

Available Expressions Algorithm

```
for all nodes n∈ℕ              // 𝔼 is set of all expressions.
   OUT[n] = 𝔼;                 // OUT[n] =𝔼 -KILL[n];
Changed = ℕ;                   // ℕ = all nodes in graph
while (Changed != ∅)
   choose n∈Changed;
   Changed=Changed-{n};
   IN[n] = 𝔼 ;
   OldOut = OUT[n]
   for all nodes p∈predecessors(n)
        IN[n]=IN[n]∩OUT[p];
   OUT[n]=(IN[n]-KILL[n])∪GEN[n];
   if (OUT[n] != OldOut)
    for all nodes s∈successors(n) Changed=Changed∪{s};
```


Questions

- Does algorithm always halt?
- If expression is available in some execution, is it always marked as available in analysis?
- If expression is not available in some execution, can it be marked as available in analysis?
- In what sense is the algorithm conservative?

Duality In Two Algorithms

- Reaching definitions
  - Confluence operation is set **union.**
  - OUT[b] initialized to **empty set**.
- Available expressions
  - Confluence operation is set **intersection.**
  - OUT[b] initialized to **set of available expressions.**
- General framework for dataflow algorithms.
- Build parameterized dataflow analyzer once, use for all dataflow problems.

Liveness Analysis

- A variable v is live at point p if
  - v is used along some path starting at p, and
  - no definition of v along the path before the use.
- When is a variable v dead at point p?
  - No use of v on any path from p to exit node, or
  - If all paths from p, redefine v before using v.

What Use is Liveness Information?

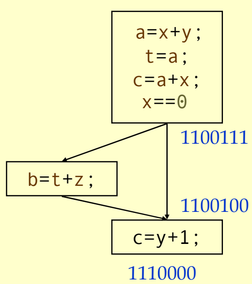
- Register allocation.
  - If a variable is dead, we can reassign its register.
- Dead code elimination.
  - Eliminate assignments to variables not read later.
  - But must not eliminate last assignment to variable (such as instance variable) visible outside CFG.
  - Can eliminate other dead assignments.
  - Handle by making all externally visible variables live on exit from CFG.

Conceptual Idea of Analysis

- Simulate execution.
- But start from exit and go **backwards** in CFG.
- Compute liveness information from end to beginning of basic blocks.

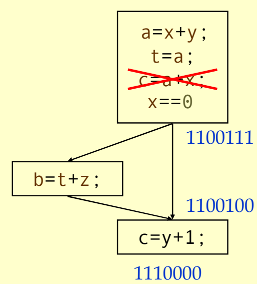
Liveness Example

- Assume `a,b,c` visible outside function. They are live on exit.
- Assume `x,y,z,t` are not visible.
- Represent liveness using a bit vector:
  order is `abcxyzt`.



Using Liveness Information for Dead Code Elimination

- Assume `a,b,c` visible outside function. They are live on exit.
- Assume `x,y,z,t` are not visible.
- Represent liveness using a bit vector:
  order is `abcxyzt`.



Formalizing Analysis

- Each basic block has
  - IN - set of variables live at start of block.
  - OUT - set of variables live at end of block.
  - USE - set of variables with upwards exposed uses in block. (**GEN**)
  - DEF - set of variables defined in block. (**KILL**)
- USE[`x=z;x=x+1;y=1;`] = {z} (x not in USE)
- DEF[`x=z;x=x+1;y=1;`] = {x, y}
- Compiler scans each basic block to derive USE and DEF sets.

Algorithm

OUT[Exit] = ∅;
IN[Exit] = USE[n];
for all nodes n∈ℕ-{Exit}
  IN[n] = ∅;
Changed = ℕ-{Exit};
while (Changed != ∅)
  choose n ∈ Changed;
  Changed = Changed-{n};
  OldIn=IN[n]
  OUT[n] = ∅;
  for all nodes s ∈ *successors*(n) OUT[n] = OUT[n] ∪ IN[p];
  IN[n] = USE[n] ∪ (OUT[n] - DEF[n]);
  if (IN[n] != OldIn)
   for all nodes p ∈ *predecessors*(n) Changed=Changed∪{p};

Similar to Other Dataflow Algorithms

Global Opt: Liveness Analysis

- Backwards analysis, not forwards.
- Still have transfer functions.
- Still have confluence operators.
- Can generalize framework to work for both forwards and backwards analyses.

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Analysis Information Inside Basic Blocks

Global Opt & BBs

- One detail:
  - Given dataflow information at IN and OUT of node.
  - Also need to compute information at each statement of basic block.
  - Simple propagation algorithm usually works fine.
  - Can be viewed as restricted case of dataflow analysis.

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Summary

Summary

- Basic blocks and basic block optimizations.
  - Copy and constant propagation.
  - Common sub-expression elimination.
  - Dead code elimination.
- Dataflow Analysis
  - Control flow graph.
  - IN[b], OUT[b], transfer functions, join points.
- Pairs of analyses and transformations:
  - Reaching definitions/constant propagation.
  - Available expressions/common sub-expression elimination.
  - Liveness analysis/Dead code elimination.

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